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Target mass and finite t corrections to diffractive deeply inelastic scattering

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Abstract

The quantum field theoretic treatment of inclusive deep–inelastic diffractive scattering given in a previous paper [1] is discussed in detail using an equivalent formulation with the aim to derive a representation suitable for data analysis. We consider the off-cone twist–2 light-cone operators to derive the target mass and finite t corrections to diffractive deep–inelastic scattering and deep–inelastic scattering. The corrections turn out to be at most proportional to $x|t|/Q^2, xM^2/Q^2$, $x = x_{\rm BJ}$ or $x_{\mathbb{P}}$, which suggests an expansion in these parameters. Their contribution varies in size considering diffractive scattering or meson–exchange processes. Relations between different kinematic amplitudes which are determined by one and the same diffractive GPD or its moments are derived. In the limit $t, M^2 \to 0$ one obtains the results of [2] and [3].

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1 Introduction

The process of deep–inelastic diffractive lepton–nucleon scattering can be measured at high energy colliders and constitutes a large fraction of the inclusive statistics, although being a semi–inclusive process. It was first observed at the electron–proton collider HERA some years ago [4] and is now measured in detail [5]. The structure function $F_2^D(x, Q^2)$ was extracted. In the same manner it is desirable to compare the longitudinal diffractive structure function $F_L^D(x, Q^2)$ with the longitudinal structure function in the inclusive case [6,7]. The measurement of the polarized diffractive structure functions $g_{1,2}^D(x, Q^2)$ will be possible at future facilities like EIC [8], which are currently planned. The experimental measurements clearly showed that the scaling violations of the deep-inelastic and the diffractive structure functions in the deep-inelastic regime, after an appropriate change of kinematic variables, are the same. Furthermore, the ratio of the two quantities, did not vary strongly, cf. [9]. While the former property is clearly of perturbative nature, the latter is of non–perturbative origin. For diffractive scattering, however, another mass scale is of importance, which is given by the invariant mass $t = (p_2 - p_1)^2$. Here $p_{1(2)}$ denote the 4–momenta of the incoming and outgoing proton, where for the latter a sufficiently large rapidity gap between this particle and the remainder final state hadrons is demanded as process signature. A similar class of processes are the so-called meson–exchange processes, cf. e.g. [10], where the finite rapidity gap is not required, but the rôle of the formerly diffractive final state proton is taken by a leading hadron, which distinguishes itself due to its high momentum from the remaining hadrons. Also in this case one may try a leading twist description, although the signature for this process is less clear than in the diffractive case.

The process of deep–inelastic diffractive scattering was first described phenomenologically [11]. A consistent field-theoretic description of the process requires factorization for the twist–2 contributions [12]. It is due to this description that reference to specific pomeron models are thoroughly avoided. In Refs. [2, 3, 13] two of the present authors gave a corresponding field–theoretic description of the process in the limit $t, M^2 \to 0$. In [2] we proved that under these conditions the scaling violations for diffractive scattering and inclusive deeply inelastic scattering are the same, up to a change in the momentum-fraction variable in the former case.

At low 4–momentum transfer Q^2 both target mass (M^2) and finite momentum transfer (t) corrections have to be considered for the diffractive and leading hadron processes with meson exchange. In the deep-inelastic case the target mass corrections were studied in Refs. [14–17], see also [18]. The kinematics of the diffractive and leading hadron processes is similar to that in deeply-virtual non–forward scattering. Considering this general class of processes, one finds that the treatment of target mass effects and finite t–effects can only be performed by combining both, see [19, 20]. If compared with the deep–inelastic case the number of hadronic structure functions enlarges in the diffractive case from two to four for unpolarized scattering and to eight for polarized scattering, as shown in [2, 3], if the general kinematics is considered. In Ref. [1] we worked out these corrections for the hadronic tensor in general, yet without quantifying the result. If one departs form the limit $t, M^2 \to 0$ the corresponding representations require to carry out a one-dimensional definite integral which kinematically relates the two proton momenta p_1 and p_2. As the integration is to be performed over unknown non-perturbative functions there is no a priori experimental way to unfold the non-perturbative distributions, which also would invalidate the partonic description in case of diffractive scattering. Moreover, the M^2 and t effects dealt with in this case are not yet complete, since there emerge other contributions more in the scattering cross section. One may expand the complete solution in two variables $t/Q^2, M^2/Q^2$. It is found that these terms multiply at least with a factor $x = x_{\mathrm{BJ}(\mathbb{P})}$, which is bounded in the diffractive case to values below 0.01 and in the meson–exchange case $\lessapprox 0.3$. Thus the leading

terms beyond $t, M^2 = 0$ give a good first estimate for the corrections. The further corrections turn out to be widely suppressed in the diffractive case, while they are larger for leading particle cross sections in the meson–exchange case.

In the present paper we will discuss both the unpolarized and polarized case. The paper is organized as follows. In Section 2 we derive the differential scattering cross section for inclusive diffractive scattering at the Lorentz level. Main aspects of the relation of this process to the Compton amplitude within the light–cone expansion including finite M^2 and t effects are summarized in Section 3. The hadronic tensors for the unpolarized and polarized case are expanded in terms of the variables t/Q^2, M^2/Q^2 in Section 4 to show the size of the correction terms. Section 5 contains the conclusions. In Appendix A we summarize some kinematic relations. The present formalism is specified to the case of deep–inelastic forward scattering (DIS) in Appendix B, where we obtain the target mass corrections given in [15–17] before.

2 The Lorentz Structure

The process of deep–inelastic diffractive scattering belongs to the class of semi–inclusive processes. It is described by an effective $2 \rightarrow 3$ diagram, cf. Figure 1 Ref. [2], with incoming and outgoing charged lepton and nucleon lines and an effective 4-vector for all the other hadron lines in the final state, which are well separated in rapidity from the outgoing diffractive nucleon line.

The differential scattering cross section for single–photon exchange is given by

$$d^5\sigma_{\rm diffr} = \frac{1}{2(s-M^2)}\frac{1}{4}\, dPS^{(3)} \sum_{\rm spins} \frac{e^4}{Q^2} L_{\mu\nu} W^{\mu\nu} \; . \tag{2.1}$$

Here $s = (p_1 + l_1)^2$ is the cms energy squared of the process and M denotes the nucleon mass. The phase space $dPS^{(3)}$ depends on five variables since the mass M_X of the diffractively produced inclusive set of hadrons varies. We choose as basic variables

$$x_{\rm BJ} = \frac{Q^2}{Q^2+W^2-M^2} = -\frac{q^2}{2\,qp_1} \; , \tag{2.2}$$

$$y = \frac{Q^2}{x_{\rm BJ}(s-M^2)} \; , \tag{2.3}$$

$t = (p_2 - p_1)^2$ the 4–momentum difference squared between incoming and outgoing nucleon, a variable describing the non-forwardness w.r.t. the incoming proton direction,

$$x_{\mathbb{P}} = \frac{Q^2+M_X^2-t}{Q^2+W^2-M^2} = -\frac{qp_-}{qp_1} \geq x_{\rm BJ} \; , \tag{2.4}$$

and the angle ϕ_b between the lepton plane $\boldsymbol{p}_1 \times \boldsymbol{l}_1$ and the hadron plane $\boldsymbol{p}_1 \times \boldsymbol{p}_2$,

$$\cos(\phi_b) = \frac{(\boldsymbol{p}_1 \times \boldsymbol{l}_1).(\boldsymbol{p}_1 \times \boldsymbol{p}_2)}{|\boldsymbol{p}_1 \times \boldsymbol{l}_1||\boldsymbol{p}_1 \times \boldsymbol{p}_2|} \; . \tag{2.5}$$

Here $Q^2 = -q^2$ denotes the photon virtuality and W is the hadronic mass with $W^2 = (p_1+q)^2$. We also refer to $x = Q^2/qp_+$. It is useful to introduce the 4–vectors

$$p_\pm = p_2 \pm p_1 \; . \tag{2.6}$$

The diffractive mass squared is given by $M_X^2 = (q - p_-)^2$. The momenta $p_\pm$ obey

$$(p_+ \, p_-) = 0, \qquad \frac{p_+^2}{p_-^2} = \frac{4M^2}{t} - 1\,. \tag{2.7}$$

For later use we refer to the non-forwardness η and the variable β defined by

$$\eta = \frac{qp_-}{qp_+} = \frac{-x_{\mathbb{P}}}{2 - x_{\mathbb{P}}} \in \left[-1\,, \frac{-x}{2-x}\right]\,, \qquad \beta = \frac{q^2}{2\,qp_-} = \frac{x_{\rm BJ}}{x_{\mathbb{P}}} \le 1\,. \tag{2.8}$$

The variable $x_{\mathbb{P}}$ is directly related to η but is more commonly used in experimental analyzes,

$$x_{\mathbb{P}} = \frac{2\eta}{\eta - 1}\,. \tag{2.9}$$

More kinematic invariants are given in Appendix A.

The transverse momentum variable, introduced as $\hat{\pi}_-$, [1], or $\pi_- = -\eta\hat{\pi}_-$ is of special importance,

$$\pi_- = p_- - p_+\eta, \qquad (q\pi_-) = 0\,. \tag{2.10}$$

Later on it plays the role of an expansion parameter. The variables $x_{\rm BJ}, x_{\mathbb{P}}, \beta$ and η obey the inequalities

$$0 \le x_{\rm BJ} \le x_{\mathbb{P}} \le 1, \qquad 0 \le x_{\rm BJ} \le \beta \le 1, \tag{2.11}$$

$$-\infty \le 1 - \frac{2}{x_{\rm BJ}} \le 1 - \frac{2\beta}{x_{\rm BJ}} = \frac{1}{\eta} \le -1 \le \eta \le \frac{-x_{\rm BJ}}{2 - x_{\rm BJ}} \le 0\,. \tag{2.12}$$

For the spin averaged cross section, the leptonic tensor is symmetric. Taking into account conservation of the electromagnetic current one obtains [2]

$$W_{\mu\nu}^s = -g_{\mu\nu}^T W_1^s + p_{1\mu}^T p_{1\nu}^T \frac{W_2^s}{M^2} + p_{2\mu}^T p_{2\nu}^T \frac{W_4^s}{M^2} + \left[p_{1\mu}^T p_{2\nu}^T + p_{2\mu}^T p_{1\nu}^T\right] \frac{W_5^s}{M^2}\,. \tag{2.13}$$

Here and in the following we do not assume that azimuthal integrals are performed as sometimes is done in experiment. In the latter case the number of contributing structure function reduces.

In the case of polarized nucleons we consider the initial state spin–vector $S_1 \equiv S,\ S^2 = -M^2$, only and sum over the spin of the outgoing hadrons. One usually refers to the longitudinal ($||$) and transverse ($\perp$) spin projections choosing

$$S_{||} = (\sqrt{E^2 - M^2}; 0, 0, 0, E)\,, \tag{2.14}$$

$$S_\perp = (0; \cos\gamma, \sin\gamma, 0) M\,, \tag{2.15}$$

in the laboratory frame with $p_1 = (E; 0, 0, \sqrt{E^2 - M^2})$, with $S.p_1 = 0$. Here γ denotes the azimuthal angle. In the case of longitudinal polarization the contraction of $S_{||}$ with l_1 and p_2 being nearly collinear to p_1 are of $\mathcal{O}(\mu^2/Q^2)$, $\mu^2 = |t|, M^2$, see Appendix A.

The antisymmetric part of the hadronic tensor was derived in [3] and is given by

$$\begin{aligned} W_{\mu\nu}^a = \; & i\left[p_{1\mu}^T p_{2\nu}^T - p_{1\nu}^T p_{2\mu}^T\right] \varepsilon_{p_1 p_2 q S} \frac{W_1^a}{M^6} + i\left[p_{1\mu}^T \varepsilon_{\nu S p_1 q} - p_{1\nu}^T \varepsilon_{\mu S p_1 q}\right] \frac{W_2^a}{M^4} \\ + \; & i\left[p_{2\mu}^T \varepsilon_{\nu S p_1 q} - p_{2\nu}^T \varepsilon_{\mu S p_1 q}\right] \frac{W_3^a}{M^4} + i\left[p_{1\mu}^T \varepsilon_{\nu S p_2 q} - p_{1\nu}^T \varepsilon_{\mu S p_2 q}\right] \frac{W_4^a}{M^4} \\ + \; & i\left[p_{2\mu}^T \varepsilon_{\nu S p_2 q} - p_{2\nu}^T \varepsilon_{\mu S p_2 q}\right] \frac{W_5^a}{M^4} + i\left[p_{1\mu}^T \varepsilon_{\nu p_1 p_2 S}^T - p_{1\nu}^T \varepsilon_{\mu p_1 p_2 S}^T\right] \frac{W_6^a}{M^4} \\ + \; & i\left[p_{2\mu}^T \varepsilon_{\nu p_1 p_2 S}^T - p_{2\nu}^T \varepsilon_{\mu p_1 p_2 S}^T\right] \frac{W_7^a}{M^4} + i\,\varepsilon_{\mu\nu q S} \frac{W_8^a}{M^2}\,, \end{aligned} \tag{2.16}$$

where $\varepsilon_{\mu\nu\alpha\beta}$ denotes the Levi–Civita symbol. The kinematic factors above are constructed out of the four–vectors q, p_1, p_2 and S as well as $g_{\mu\nu}$ and $\varepsilon_{v_0 v_1 v_2 v_3}$ using

$$
\begin{aligned}
p_\mu^T &= p_\mu - q_\mu \frac{q.p}{q^2}, \quad g_{\mu\nu}^T = g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}, && (2.17) \\
\varepsilon_{\mu v_1 v_2 v_3}^T &= \varepsilon_{\mu v_1 v_2 v_3} - \varepsilon_{q v_1 v_2 v_3} \frac{q_\mu}{q^2}, && (2.18) \\
\varepsilon_{\mu\nu v_1 v_2}^{TT} &= \varepsilon_{\mu\nu v_1 v_2} - \varepsilon_{q\nu v_1 v_2} \frac{q_\mu}{q^2} - \varepsilon_{\mu q v_1 v_2} \frac{q_\nu}{q^2}. && (2.19)
\end{aligned}
$$

One may rewrite (2.16) into an equivalent form using the Schouten-identities [21].

Target mass and finite t corrections to the differential scattering cross section (2.1) in the leading twist approximation emerge from three sources: (i) from kinematic terms at the Lorentz level after contracting the leptonic and hadronic tensor; (ii) from the expectation value of the Compton operator; (iii) the t-behaviour of the non–perturbative distribution functions.

We will first consider the contributions (i) and discuss the terms (ii) in Section 4. The non-perturbative effects cannot be calculated by rigorous methods within Quantum Chromodynamics at present, but are left to phenomenological models or are determined through fits to data, cf. [11].

For pure photon exchange the leptonic tensor is given by

$$
L_{\mu\nu} = 2(l_{1\mu} l_{2\nu} + l_{2\mu} l_{1\nu} - g_{\mu\nu} l_1.l_2 - i\varepsilon_{\mu\nu\alpha\beta} l_1^\alpha q^\beta), \tag{2.20}
$$

cf. [22], in case of longitudinal lepton polarization.

We consider the Bjorken limit,

$$
2p_1.q = 2M\nu \rightarrow \infty, \quad p_2.q \rightarrow \infty, \quad Q^2 \rightarrow \infty, \quad \text{with} \quad x_{\rm BJ} \quad \text{and} \quad x_{\mathbb{P}} = \text{fixed}. \tag{2.21}
$$

Here,

$$
\begin{aligned}
M W_1^s &\rightarrow F_1 && (2.22) \\
\nu W_k^s &\rightarrow F_k, \quad k = 2, 4, 5, && (2.23)
\end{aligned}
$$

with $\nu = y(s - M^2)/(2M)$.

In the unpolarized case we obtain in the limit $M^2, t \rightarrow 0$ w.r.t. the kinematics of the momenta p_1 and p_2, keeping the target mass dependence

$$
\frac{d^s \sigma^{\rm unpol}}{dx_{\rm BJ}\, dQ^2} = \frac{2\pi\alpha^2}{Q^4 x_{\rm BJ}} \left[2xF_1 \cdot y^2 + \left[F_2 + (1 - x_{\mathbb{P}})F_4 + (1 - x_{\mathbb{P}})^2 F_5\right] \cdot 2\left(1 - y - \frac{x_{\rm BJ}^2 y^2 M^2}{Q^4}\right)\right] \tag{2.24}
$$

where $F_k = F_k(x_{\rm BJ}, x_{\mathbb{P}}, Q^2; t)$ are the diffractive structure functions, cf. [2]. The correction terms are of $\mathcal{O}(M^2/Q^2,\ t/Q^2)$. In the limit $M^2, t \rightarrow 0$ the azimuthal dependence on ϕ_b vanishes.

Likewise we obtain in the polarized case for longitudinal nucleon polarization,

$$
\begin{aligned}
\frac{d^3\sigma^{\rm pol}(\lambda, \pm S_{||})}{dx_{\rm BJ} dQ^2 dx_{\mathbb{P}}} &= \mp 4\pi s\lambda \frac{\alpha^2}{Q^4} \left[y\left(2 - y - \frac{2x_{\rm BJ} y M^2}{s}\right) x g_1 - 4x_{\rm BJ} y \frac{M^2}{s} g_2 \right], && (2.25) \\
\frac{d^4\sigma^{\rm pol}(\lambda, \pm S_\perp)}{dx_{\rm BJ} dQ^2 dx_{\mathbb{P}}.d\Phi} &= \mp 4\pi s\lambda \sqrt{\frac{M^2}{s}} \frac{\alpha^2}{Q^2} \sqrt{x_{\rm BJ} y \left[1 - y - \frac{x_{\rm BJ} y M^2}{s}\right]} \cos(\gamma - \Phi) \left[y x_{\rm BJ} g_1 + 2x_{\rm BJ} g_2\right]. && (2.26)
\end{aligned}
$$

Here ϕ denotes the angle between the $\vec{l}_1 - \vec{S}$ and the $\vec{l}_1 - \vec{l}_2$ plane and α is the anle between $\vec{l}_1$ and $\vec{S}$. The structure functions $g_{1,2}(x_{\rm BJ}, x_{\mathbb{P}}, Q^2; t)$ are obtained from $W_2^a, W_3^a, W_4^a, W_5^a$ and W_8^a

by

$$g_1 = \frac{p.q_1}{M^2} W_8^a \ , \tag{2.27}$$

$$g_2 = \frac{(p.q_1)^3}{q^2 M^4} \left[W_2^a + (1-x_{\mathbb{P}})[W_3^a + W_4^a] + (1-x_{\mathbb{P}})^2 W_5^a\right] \tag{2.28}$$

and the different structure functions F_i and g_i depend on the variables $x_{\rm BJ}, x_{\mathbb{P}}, Q^2$ and t.

3 The Compton Amplitude

The hadronic tensor for deep–inelastic diffractive scattering can be obtained from a Compton amplitude as has been outlined in [1–3] before. We limit the description to the level of the twist–2 contributions, where factorization holds for the semi–inclusive diffractive process [12]. Furthermore, A. Mueller's generalized optical theorem [23] allows to move the final state proton into an initial state anti-proton, where both particle momenta are separated by t and form a formal 'quasi two–particle' state $|p_1, -p_2, S; t\rangle$. These states are used to form the operator matrix elements. The correctness of this procedure within the light–cone expansion relies, first, on the rapidity gap between the outgoing proton and the remaining hadronic part with invariant mass M_X and, second, on the special property of matrix elements of the contributing light-cone operators to contain no absorptive part. Independently, one could argue that the corresponding matrix element is a pure phenomenological quantity satisfying restrictions imposed by quantum field theory. The general structure of the scattering amplitude is completely determined by the off-cone structure of the twist-2 Compton operator (3.4), cf. [24].

The structure functions for the diffractive process can thus be obtained by analyzing the absorptive part

$$W_{\mu\nu} = \mathsf{Im} T_{\mu\nu} \tag{3.1}$$

of the expectation value

$$T_{\mu\nu}(x) = \langle p_1, -p_2, S; t| \; \widehat{T}_{\mu\nu}(x) \, |p_1, -p_2, S; t\rangle \, , \tag{3.2}$$

with the well-known operator $\widehat{T}_{\mu\nu}$ of (virtual) Compton scattering defined as

$$\widehat{T}_{\mu\nu}(x) \equiv iRT \left[J_\mu \left(\frac{x}{2}\right) \; J_\nu \left(-\frac{x}{2}\right) \mathcal{S} \right] \, . \tag{3.3}$$

In [1], based on a general quantum field theoretic consideration of virtual Compton scattering at twist 2 [20, 25, 26], we specified the various terms which contribute to the general structure of the hadronic tensor $W_{\mu\nu} = \mathsf{Im} T_{\mu\nu}$ in case of deep–inelastic diffractive scattering. As shown in [27, 28] the operator $\widehat{T}_{\mu\nu}$ in lowest order of the non–local light–cone expansion [29] contains the vector or axial vector operators only. The scattering amplitude is obtained by the Fourier transform of the operator $\widehat{T}_{\{\mu\nu\}}(x)$ and forming the matrix element (3.2). Here, we want to study its twist–2 contributions including target mass and finite momentum transfer corrections. This is obtained by harmonic extension [26, 30, 31] of the twist–2 light-cone operators to twist–2 off-cone operators [32], leading to

$$\widehat{T}^{\rm tw2}_{\mu\nu}(q) = -e^2 \int \frac{{\rm d}^4 x}{2{\rm i}\pi^2} \frac{{\rm e}^{{\rm i}qx}\, x_\lambda}{(x^2 - {\rm i}\epsilon)^2} \left\{ S_{\mu\nu|}{}^{\alpha\lambda}\, O^{\rm tw2}_\alpha(\kappa x, -\kappa x) + \epsilon_{\mu\nu}{}^{\alpha\lambda}\, O^{\rm tw2}_{5\,\alpha}(\kappa x, -\kappa x) \right\} \, , \tag{3.4}$$

with

$$
\begin{aligned}
O_{\alpha}^{\rm tw2}(\kappa x,-\kappa x) &= i[\overline{\psi}(\kappa x)\gamma_\alpha\psi(-\kappa x) - [\overline{\psi}(-\kappa x)\gamma_\alpha\psi(\kappa x)]^{\rm tw2}, \\
O_{5\,\alpha}^{\rm tw2}(\kappa x,-\kappa x) &= [\overline{\psi}(\kappa x)\gamma_5\gamma_\alpha\psi(-\kappa x) + [\overline{\psi}(-\kappa x)\gamma_5\gamma_\alpha\psi(\kappa x)]^{\rm tw2},
\end{aligned}
$$

and $\kappa = 1/2$. The matrix elements can be written in terms of vectors $\mathcal{K}^a_{\mu,(5)}$ and 2–dimensional Fourier-integrals over partonic distributions $f_{a(5)}(z_+,z_-,t)$ summing over a,

$$
\langle p_1,-p_2;t|\, e^2\, O_{\mu}^{\rm tw2}(\kappa x,-\kappa x)\,|p_1,-p_2;t\rangle = \mathcal{K}^a_{\mu}(p_\pm)\int \frac{D\mathbb{Z}}{(2\pi)^4}\, e^{i\kappa x(p_-z_-+p_+z_+)}\, f_a(z_+,z_-,t)\,, \tag{3.5}
$$

$$
\langle p_1,-p_2,S;t|\, e^2\, O_{5\mu}^{\rm tw2}(\kappa x,-\kappa x)\,|p_1,-p_2,S;t\rangle = \mathcal{K}^a_{5\,\mu}(p_\pm,S)\int \frac{D\mathbb{Z}}{(2\pi)^4}\, e^{i\kappa x(p_-z_-+p_+z_+)}\, f_{5\,a}(z_+,z_-,t)\,, \tag{3.6}
$$

which is defined as asymptotic expression on the light-cone at $x^2 = 0$.

We choose as kinematic factors for the representation of the matrix element of the non-local operator for the symmetric part (3.5)

$$
\mathcal{K}^{1\,\mu} = p_+^\mu\,, \qquad\qquad \mathcal{K}^{2\,\mu} = \pi_-^\mu \equiv p_-^\mu - \eta p_+^\mu\,, \tag{3.7}
$$

and for its antisymmetric part (3.6)

$$
\mathcal{K}_5^{1\,\mu} = S^\mu\,, \qquad \mathcal{K}_5^{2\,\mu} = p_+^\mu\,(p_2S)/M^2\,, \qquad \mathcal{K}_5^{3\,\mu} = \pi_-^\mu\,(p_2S)/M^2\,. \tag{3.8}
$$

The normalization to M^2 in (3.8) is arbitrary and has to be arranged with the definition of the corresponding distribution functions $f_{a,(5\,a)}(z_+,z_-)$, respectively. The corresponding Lorentz-invariant has to be formed out of the hadronic momenta, except the spin vector, since the polarization–symmetries are assumed to be linear in the spin.

The momentum fractions $z_\pm$ in (3.5, 3.6) corresponding to the momenta $p_\pm$ are

$$
\mathbb{P} = (p_+,p_-) = (p_2+p_1, p_2-p_1), \quad \mathbb{Z} = (z_+,z_-) = ((z_2+z_1)/2, (z_2-z_1)/2)\,, \tag{3.9}
$$

with the measure $D\mathbb{Z}$

$$
D\mathbb{Z} = 2\, dz_+dz_-\, \theta(1-z_++z_-)\,\theta(1+z_+-z_-)\,\theta(1-z_+-z_-)\,\theta(1+z_++z_-)\,. \tag{3.10}
$$

We refer to $f_{a(5)}(z_+,z_-,t)$ as diffractive generalized parton distribution functions, (dGPD), in distinction to the GPDs emerging in deeply virtual Compton scattering [33]. These amplitudes are directly connected to the total cross sections and polarization asymmetries, respectively. Both kinds of GPDs are expectation values of the same light–cone operator, however, between different states. Interesting limiting cases can be derived from them. For the dGPDs these are the quasi collinear limit: $\pi_- \to 0, M^2 \to 0$, [2, 3], and the limit of deep–inelastic scattering, see Appendix B. Furthermore, for both types of GPDs the evolution equations are derived from the renormalization group equation for the same light–cone operators. It is remarkable, that the evolution equations for the dGPDs are two–variable equations which reduce to the simple evolution equation for forward scattering in the quasi collinear limit, cf. [2].

The (dimensionless) amplitudes $f_{(5)\,a}(z_+,z_-,t)$ depend on t and η explicitly. In addition, there appears a t– and M^2–dependence of the amplitude (3.2) in momentum space, which finally, on the one hand, results from the Fourier transform in (3.4) where the operator $O^{\rm tw2}_{(5)\,\alpha}(\kappa x,-\kappa x)$ is

off the light-cone, i.e. with all trace subtractions. On the other hand, the dependence results from the kinematic pre–factors $\mathcal{K}^a_{(5)\,\mu}(p_\pm, S)$.[1]

Concerning the independent kinematic factors one has two possibilities, which are mathematically equivalent, depending on whether one chooses p_- or p_+ as essential variable as we did in our previous papers [1] and [20], respectively. The corresponding choices lead to different dGPDs.

(1) In the first case, which we considered in [1], cf. also [2] and [3], p_- was chosen as essential variable, by starting from the physical picture using the generalized optical theorem, and the parameterization [2]

$$p_- z_- + p_+ z_+ \;= \hat\lambda\,[p_- + \hat\zeta(p_+ - p_-/\eta)] = \hat\lambda\,[p_- + \hat\zeta\hat\pi_-] \equiv \hat\lambda\,\hat{\mathcal{P}}, \tag{3.11}$$

with

$$\begin{aligned} \hat\lambda &= z_- + z_+/\eta\ , \\ z_+ &= \hat\lambda\,\hat\zeta\ , \\ z_- &= \hat\lambda\,(1-\hat\zeta/\eta)\ . \end{aligned} \tag{3.12}$$

(2) A mathematically equivalent description is obtained starting from p_+ as the essential variable [20]. In this approach we introduce the new variables λ and ζ instead of z_+ and z_-,

$$p_- z_- + p_+ z_+ = \lambda\,[\,p_+ + \zeta\,(p_- - \eta\,p_+)] = \lambda\,(p_+ + \zeta\,\pi_-) \equiv \lambda\,\mathcal{P} = 2\,\Pi\,, \tag{3.13}$$

with

$$\begin{aligned} \lambda &= z_+ + \eta z_-\ , \\ z_- &= \lambda\zeta\ , \\ z_+ &= \lambda(1-\zeta\eta)\ . \end{aligned} \tag{3.14}$$

Here the variable λ plays the role of a common scale for $z_\pm$. Compared to Ref. [20] we list the essential kinematic variables using the above parameterization

$$\mathcal{P}(\eta,\zeta) = p_+(1-\eta\,\zeta) + p_-\zeta\ , \tag{3.15}$$

$$\mathcal{P}^2 = p_+^2 - 2\,\zeta\,\eta p_+^2 + \zeta^2(p_-^2 + p_+^2\eta^2), \tag{3.16}$$

$$q\mathcal{P} = qp_+,$$

$$\mathcal{P}^2/(\mathcal{P}^{\rm T})^2 = x^2(\mathcal{P}^2/Q^2)\,/\,\big[1 + x^2(\mathcal{P}^2/Q^2)\big]\,, \tag{3.17}$$

and

$$\xi_\pm = \frac{2x}{1 \pm \sqrt{1 + x^2\mathcal{P}^2/Q^2}}\,, \qquad x = \frac{Q^2}{qp_+} = \frac{Q^2}{2qp_1}(1-\eta) = x_{\rm BJ}(1-\eta) = -2\,\beta\,\eta. \tag{3.18}$$

Obviously, $\xi_+ \equiv \xi$ is the appropriate generalization of the Nachtmann variable. With these definitions the measure of the $\mathbb{Z}$–integration is

$$\begin{aligned} D\mathbb{Z} = 2|\lambda|\,d\lambda\,d\zeta\,&\theta\big(1 - \lambda + (1+\eta)\lambda\,\zeta\big)\,\theta\big(1 + \lambda - (1+\eta)\lambda\,\zeta\big) \\ &\times\theta\big(1 - \lambda - (1-\eta)\lambda\,\zeta\big)\,\theta\big(1 + \lambda + (1-\eta)\lambda\,\zeta\big)\ . \end{aligned} \tag{3.19}$$

[1] In the following the explicit t–dependence of the distribution functions is always understood and we drop this variable to lighten the notation.

[2] For later convenience the notation (ϑ, ζ) of Ref. [1] has been changed into $(\hat\lambda, \hat\zeta)$.

In the present treatment we choose p_+ as the essential variable.

In Ref. [1] deep–inelastic diffractive scattering has been worked out within the first approach. The resulting expressions contain an internal $\hat{\zeta}$–integral which is not well suited for the direct comparison of experimental data with the diffractive GPDs. One way out is to introduce new 'integrated distributions'. Furthermore, we can perform a systematic $1/Q^2$ expansion which leads to an expansion in terms of $\mathcal{P}^2/Q^2$ directly. Since $\mathcal{P}^2$ is a polynomial of second order in the variable ζ we are led to a ζ– expansion,

$$\begin{aligned} \hat{\mathcal{P}}^2 &= t - 2\hat{\zeta}\, t/\eta + (4M^2 - t + t/\eta^2)\,\hat{\zeta}^2|_{\hat{\zeta}\to 0} = t\,, \qquad (3.20)\\ \mathcal{P}^2 &= \hat{\mathcal{P}}^2/\,\eta^2 = (4\,M^2 - t)(1 - 2\,\eta\,\zeta) + \left[t + (4\,M^2 - t)\,\eta^2\right]\zeta^2|_{\zeta\to 0} = (4M^2 - t). \qquad (3.21) \end{aligned}$$

We prefer the second parameterization which leads to expressions which contain as lowest approximation the mass corrections known from deep–inelastic scattering, without requiring any further redefinition of the dGPDs. We use the original expression for the Compton scattering amplitude [20] with the λ–parameterization and apply the matrix elements (3.5, 3.6).

4 The Hadronic Tensor

In the following we discuss the symmetric and antisymmetric contributions to the hadronic tensor, which correspond to the unpolarized and polarized case, separately.

4.1 The Symmetric Part

The symmetric part of the hadronic tensor for diffractive scattering, cf. [1, 20] is given by

$$\begin{aligned} W^{\rm tw2}_{\{\mu\nu\}}(q) &= \mathrm{Im}\, \frac{q^2}{2}\int D\mathbb{Z}\, \frac{\mathcal{A}_{\{\mu\nu\}}(q,\mathcal{P})}{\lambda\sqrt{(q\mathcal{P})^2 - q^2\mathcal{P}^2}}\left(\frac{1}{1-\xi_+/\lambda + i\varepsilon} - \frac{1}{1-\xi_-/\lambda - i\varepsilon}\right)\\ &= -2\pi\int d\zeta \frac{q^2}{\sqrt{(q\mathcal{P})^2 - q^2\mathcal{P}^2}}\Bigg\{\frac{q\mathcal{K}^a}{q\mathcal{P}}\left[g^{\rm T}_{\mu\nu}F_{a\,1}(\xi,\zeta) - \frac{\mathcal{P}^{\rm T}_\mu\mathcal{P}^{\rm T}_\nu}{(\mathcal{P}^{\rm T})^2}F_{a\,2}(\xi,\zeta)\right]\\ &\qquad + \left(\frac{q\mathcal{K}^a}{q\mathcal{P}} - \frac{\mathcal{P}\mathcal{K}^a}{\mathcal{P}^2}\right)\left[g^{\rm T}_{\mu\nu}F_{a\,3}(\xi,\zeta) - \frac{\mathcal{P}^{\rm T}_\mu\mathcal{P}^{\rm T}_\nu}{(\mathcal{P}^{\rm T})^2}F_{a\,4}(\xi,\zeta)\right]\\ &\qquad - \left(\frac{\mathcal{K}^{a\,\rm T}_\mu\mathcal{P}^{\rm T}_\nu + \mathcal{P}^{\rm T}_\mu\mathcal{K}^{a\,\rm T}_\nu}{(\mathcal{P}^{\rm T})^2} - 2\,\frac{q\mathcal{K}^a}{q\mathcal{P}}\,\frac{\mathcal{P}^{\rm T}_\mu\mathcal{P}^{\rm T}_\nu}{(\mathcal{P}^{\rm T})^2}\right)F_{a\,5}(\xi,\zeta)\Bigg\}. \end{aligned} \qquad (4.1)$$

The relevant imaginary part belongs to the δ–distribution $\delta(1 - \xi_+/\lambda)$ in terms of variables $(\xi_+ \equiv \xi, \zeta)$, with the λ–integration, (3.19), being carried out. It implies the pole condition, cf. [20] Eqs. (6.6–6.10) and [1],

$$1 + \tfrac{1}{2}\xi\, x\,\mathcal{P}^2/Q^2 = \sqrt{1 + x^2\mathcal{P}^2/Q^2} = -(1 - 2\,x/\,\xi)\;. \qquad (4.2)$$

which we use below. The structure functions $F_{ai}, i = 1, \ldots, 5$ are given by

$$F_{a\,1}(\xi,\,\zeta) = \Phi^{(0)}_a(\xi,\zeta) + \frac{1}{2}\frac{x\,\mathcal{P}^2/Q^2}{\sqrt{1 + x^2\mathcal{P}^2/Q^2}}\,\Phi^{(1)}_a(\xi,\zeta) + \frac{1}{4}\frac{(x\,\mathcal{P}^2/Q^2)^2}{1 + x^2\mathcal{P}^2/Q^2}\,\Phi^{(2)}_a(\xi,\zeta)\,, \qquad (4.3)$$

$$F_{a\,2}(\xi,\,\zeta) = \Phi^{(0)}_a(\xi,\zeta) + \frac{3}{2}\frac{x\,\mathcal{P}^2/Q^2}{\sqrt{1 + x^2\mathcal{P}^2/Q^2}}\,\Phi^{(1)}_a(\xi,\zeta) + \frac{3}{4}\frac{(x\,\mathcal{P}^2/Q^2)^2}{1 + x^2\mathcal{P}^2/Q^2}\,\Phi^{(2)}_a(\xi,\zeta)\,. \qquad (4.4)$$

$$
\begin{aligned}
F_{a\,3}(\xi,\zeta) &= -\frac{1}{2}\frac{\xi\, x\,\mathcal{P}^2/Q^2}{\sqrt{1+x^2\mathcal{P}^2/Q^2}}\,\Phi_a^{(0)}(\xi,\zeta) \qquad (4.5)\\
&+\frac{1}{2\xi}\left(\frac{\xi\, x\,\mathcal{P}^2/Q^2}{\sqrt{1+x^2\mathcal{P}^2/Q^2}}-\frac{(\xi\, x\,\mathcal{P}^2/Q^2)^2}{1+x^2\mathcal{P}^2/Q^2}\right)\Phi_a^{(1)}(\xi,\zeta)\\
&-\frac{1}{\xi}\left(\frac{\xi\, x\,\mathcal{P}^2/Q^2}{\sqrt{1+x^2\mathcal{P}^2/Q^2}}-\frac{(\xi\, x\mathcal{P}^2/Q^2)^2}{1+x^2\mathcal{P}^2/Q^2}+\frac{3}{8}\frac{(\xi\, x\,\mathcal{P}^2/Q^2)^3}{\sqrt{1+x^2\mathcal{P}^2/Q^2}^{\,3}}\right)\int_\xi^1\frac{dy}{y}\,\Phi_a^{(1)}(y,\zeta)\\
&-\frac{1}{\xi}\left(\frac{(\xi\, x\,\mathcal{P}^2/Q^2)^2}{1+x^2\mathcal{P}^2/Q^2}-\frac{3}{4}\frac{(\xi\, x\,\mathcal{P}^2/Q^2)^3}{\sqrt{1+x^2\mathcal{P}^2/Q^2}^{\,3}}+\frac{3}{16}\frac{(\xi\, x\,\mathcal{P}^2/Q^2)^4}{[1+x^2\mathcal{P}^2/Q^2]^2}\right)\int_\xi^1\frac{dy}{y^2}\,\Phi_a^{(2)}(y,\zeta)\,,\\
F_{a\,4}(\xi,\zeta) &= -\frac{1}{2}\frac{\xi\, x\mathcal{P}^2/Q^2}{\sqrt{1+x^2\mathcal{P}^2/Q^2}}\,\Phi_a^{(0)}(\xi,\zeta) \qquad (4.6)\\
&+\frac{1}{\xi}\left(\frac{5}{2}\frac{\xi\, x\mathcal{P}^2/Q^2}{\sqrt{1+x^2\mathcal{P}^2/Q^2}}-\frac{3}{2}\frac{(\xi\, x\mathcal{P}^2/Q^2)^2}{1+x^2\mathcal{P}^2/Q^2}\right)\Phi_a^{(1)}(\xi,\zeta)\\
&-\frac{3}{\xi}\left(\frac{\xi\, x\mathcal{P}^2/Q^2}{\sqrt{1+x^2\mathcal{P}^2/Q^2}}-2\frac{(\xi\, x\mathcal{P}^2/Q^2)^2}{1+x^2\mathcal{P}^2/Q^2}+\frac{5}{8}\frac{(\xi\, x\mathcal{P}^2/Q^2)^3}{\sqrt{1+x^2\mathcal{P}^2/Q^2}^{\,3}}\right)\int_\xi^1\frac{dy}{y}\,\Phi_a^{(1)}(y,\zeta)\\
&-\frac{3}{\xi}\left(\frac{(\xi\, x\mathcal{P}^2/Q^2)^2}{1+x^2\mathcal{P}^2/Q^2}-\frac{5}{4}\frac{(\xi\, x\mathcal{P}^2/Q^2)^3}{\sqrt{1+x^2\mathcal{P}^2/Q^2}^{\,3}}+\frac{5}{16}\frac{(\xi\, x\mathcal{P}^2/Q^2)^4}{[1+x^2\mathcal{P}^2/Q^2]^2}\right)\int_\xi^1\frac{dy}{y^2}\,\Phi_a^{(2)}(y,\zeta)\,,\\
F_{a\,5}(\xi,\zeta) &= \frac{1}{\xi}\Bigg[\Phi_a^{(1)}(\xi,\zeta)+\frac{3}{2}\frac{\xi\, x\mathcal{P}^2/Q^2}{\sqrt{1+x\mathcal{P}^2/Q^2}}\int_\xi^1\frac{dy}{y}\,\Phi_a^{(1)}(y,\zeta)\\
&+\frac{3}{4}\frac{(\xi\, x\mathcal{P}^2/Q^2)^2}{1+x\mathcal{P}^2/Q^2}\int_\xi^1\frac{dy}{y^2}\,\Phi_a^{(2)}(y,\zeta)\Bigg]. \qquad (4.7)
\end{aligned}
$$

Whereas $F_{a\,1(2)}(\xi,\zeta)$ are direct generalizations of the well-known deep–inelastic structure functions. $F_{a\,k}(\xi,\zeta)|_{k=3,4,5}$ are new structure functions, which vanish in the forward limit, cf. Appendix B. The typical square roots $\sqrt{1+x^2\mathcal{P}^2/Q^2}$ for the mass corrections depend on the generalized momentum $\mathcal{P}=\mathcal{P}(\zeta)$. After substituting $\lambda\to\xi$ in (4.3–4.7), we introduce the following iterated representations for the basic dGPDs $f_a(\lambda,\zeta)$, cf. (3.5):

$$
\Phi_a^{(0)}(\xi,\zeta)\equiv f_a(\xi,\zeta)\,, \qquad (4.8)
$$

$$
\Phi_a^{(1)}(\xi,\zeta)\equiv\int_\xi^1 dy_1\; f_a(y_1,\zeta)=\xi\int_0^1\frac{d\tau}{\tau^2}\;f_a\Big(\frac{\xi}{\tau},\zeta\Big)\,, \qquad (4.9)
$$

$$
\Phi_a^{(2)}(\xi,\zeta)\equiv\int_\xi^1 dy_2\int_{y_2}^1 dy_1\, f_a(y_1,\zeta)=\xi^2\int_0^1\frac{d\tau_1}{\tau_1^3}\int_0^1\frac{d\tau_2}{\tau_2^2}\;f_a\Big(\frac{\xi}{\tau_1\tau_2},\zeta\Big)\,, \qquad (4.10)
$$

$$
\Phi_a^{(i)}(\xi,\zeta)\equiv\int_\xi^1 dy\;\Phi_a^{(i-1)}(y,\zeta)\,,\qquad \text{for}\quad i\geq 1\,, \qquad (4.11)
$$

$$
\int_\xi^1\frac{dy}{y}\,\Phi_a^{(1)}(y,\zeta)\equiv\int_\xi^1\frac{dy_1}{y_1}\int_{y_1}^1 dy\,\Phi_a^{(0)}(y,\zeta)=\xi\int_0^1\frac{d\tau_1}{\tau_1^2}\int_0^1\frac{d\tau_2}{\tau_2^2}\;f_a\Big(\frac{\xi}{\tau_1\tau_2},\zeta\Big)\,, \qquad (4.12)
$$

$$
\int_\xi^1\frac{dy}{y^2}\,\Phi_a^{(2)}(y,\zeta)\equiv\int_\xi^1\frac{dy_1}{y_1^2}\int_{y_1}^1 dy\,\Phi_a^{(1)}(y,\zeta)=\xi\int_0^1\frac{d\tau_1}{\tau_1^3}\int_0^1\frac{d\tau_2}{\tau_2^2}\int_0^1\frac{d\tau_3}{\tau_3^2}\;f_a\Big(\frac{\xi}{\tau_1\tau_2\tau_3},\zeta\Big). \qquad (4.13)
$$

Let us now investigate the effect of target masses and finite terms in t in more detail. It turns out that both the M^2– and t–contributions in the diffractive structure functions emerge due to

the parameter ρ

$$\rho = \epsilon\, x^2 \frac{p_+^2}{Q^2} \frac{1}{1 + x^2 p_+^2/Q^2} \;, \tag{4.14}$$

with ϵ given by $\mathcal{P}^2 = p_+^2(1+\epsilon)$,

$$\epsilon = \frac{1}{p_+^2}\left[2\zeta p_+ \pi_- + \zeta^2 \pi_-^2\right] = -2\eta\zeta + \left(\eta^2 + \frac{t}{p_+^2}\right)\zeta^2 \;. \tag{4.15}$$

Since

$$-\,\eta \simeq x_{\mathbb{P}} \;\ll 1 \;, \tag{4.16}$$

ρ effectively takes values $\rho \lesssim 10^{-3}$ for $x_{\mathbb{P}} \lesssim 10^{-2}, |t| \approx (0.1...1)M^2, Q^2 \approx (1...5)M^2$. The range of ζ is determined both by the support condition (3.19) and the condition $\mathcal{P}^2 = p_+^2(1+\epsilon) > 0$ in the diffractive case.

To prepare the expansion in ρ we rewrite the hadronic tensor as

$$\begin{aligned} W^{\rm tw2}_{\{\mu\nu\}}(q) = 2\pi \int d\zeta\, \frac{q\mathcal{K}^a}{q\mathcal{P}} & \left[-\, g^{\rm T}_{\mu\nu}\, W^{\rm diff}_{a\,1}\Big(x, \frac{\mathcal{P}^2}{Q^2}; \zeta\Big) + \frac{\mathcal{P}^{\rm T}_\mu \mathcal{P}^{\rm T}_\nu}{M^2}\, W^{\rm diff}_{a\,2}\Big(x, \frac{\mathcal{P}^2}{Q^2}; \zeta\Big) \right], \\ & + \left\{ \left(\frac{q\mathcal{K}^a}{q\mathcal{P}} - \frac{\mathcal{P}\mathcal{K}^a}{\mathcal{P}^2} \right) \frac{\mathcal{P}^2}{Q^2} \left[-\, g^{\rm T}_{\mu\nu} W^{\rm diff}_{a\,3}\Big(x, \frac{\mathcal{P}^2}{Q^2}; \zeta\Big) + \frac{\mathcal{P}^{\rm T}_\mu \mathcal{P}^{\rm T}_\nu}{M^2} W^{\rm diff}_{a\,4}\Big(x, \frac{\mathcal{P}^2}{Q^2}; \zeta\Big) \right] \right. \\ & \left. + \left(\mathcal{P}^{\rm T}_\mu \mathcal{K}^{a\,\rm T}_\nu + \mathcal{P}^{\rm T}_\nu \mathcal{K}^{a\,\rm T}_\mu - 2\, \mathcal{P}^{\rm T}_\mu \mathcal{P}^{\rm T}_\nu \frac{q\mathcal{K}^a}{q\mathcal{P}} \right) \frac{1}{M^2} W^{\rm diff}_{a\,5}\Big(x, \frac{\mathcal{P}^2}{Q^2}; \zeta\Big) \right\} \;. \end{aligned} \tag{4.17}$$

The integral over ζ cannot be performed easily. Here, the un-integrated structure functions $W^{\rm diff}_{a\,k}(x, \mathcal{P}^2(\zeta)/Q^2; \zeta)$ are given by

$$W^{\rm diff}_{a\,1}\Big(x, \frac{\mathcal{P}^2(\zeta)}{Q^2}; \zeta\Big) \equiv -\frac{x}{\sqrt{1 + x^2\mathcal{P}^2/Q^2}}\; F_{a\,1}(\xi,\zeta) \;, \tag{4.18}$$

$$W^{\rm diff}_{a\,3}\Big(x, \frac{\mathcal{P}^2(\zeta)}{Q^2}; \zeta\Big) \equiv -\frac{x}{\sqrt{1 + x^2\mathcal{P}^2/Q^2}} \frac{Q^2}{\mathcal{P}^2}\; F_{a\,3}(\xi,\zeta) \;, \tag{4.19}$$

$$W^{\rm diff}_{a\,k}\Big(x, \frac{\mathcal{P}^2(\zeta)}{Q^2}; \zeta\Big) \equiv -\frac{M^2}{Q^2} \left(\frac{x}{\sqrt{1 + x^2\mathcal{P}^2/Q^2}} \right)^3 F_{a\,k}(\xi,\zeta) \qquad \text{for } k = 2, 5 \;, \tag{4.20}$$

$$W^{\rm diff}_{a\,4}\Big(x, \frac{\mathcal{P}^2(\zeta)}{Q^2}; \zeta\Big) \equiv -\frac{M^2}{\mathcal{P}^2} \left(\frac{x}{\sqrt{1 + x^2\mathcal{P}^2/Q^2}} \right)^3 F_{a\,4}(\xi,\zeta) \;. \tag{4.21}$$

As noted in [1] a generalized Callan–Gross [34] relation between $W^{\rm diff}_{a\,1}$ and $W^{\rm diff}_{a\,2}$, which holds for diffractive scattering in the limit $M^2,\ t\ \to\ 0$, [2], is broken as in the case of deep–inelastic scattering [15]. Correspondingly, the distribution functions $W^{\rm diff}_{a\,(1,2)}$ are related to $W^{\rm diff}_{a\,\rm L}$, the diffractive analogue of the longitudinal structure function of deep–inelastic scattering, by

$$W^{\rm diff}_{a\,\rm L}(x, \mathcal{P}^2/Q^2; \zeta) = -\, W^{\rm diff}_{a\,1}(x, \mathcal{P}^2/Q^2; \zeta) + \left(1 + \frac{x^2\mathcal{P}^2}{Q^2}\right) \frac{qp_+}{x\, M^2}\, W^{\rm diff}_{a\,2}(x, \mathcal{P}^2/Q^2; \zeta) \,. \tag{4.22}$$

To see this in detail, we insert (4.3), (4.4) and (4.20), so that

$$W^{\rm diff}_{a\,\rm L}(x, \mathcal{P}^2/Q^2; \zeta) = \frac{x}{\sqrt{1 + x^2\mathcal{P}^2/Q^2}} \big(F_{a\,1}(\xi,\zeta) - F_{a\,2}(\xi,\zeta)\big) \approx \mathcal{O}\left(\frac{x^2\mathcal{P}^2}{Q^2}\right) . \tag{4.23}$$

The last relation follows by direct inspection of $F_{a\,i}$ and is explicit in

$$W^{\rm diff}_{a\,\rm L}(x,\mathcal{P}^2/Q^2;\zeta) = -\frac{x^2\mathcal{P}^2}{2Q^2}\frac{\partial}{\partial x}\Big(\frac{x}{\xi\sqrt{1+x^2\mathcal{P}^2/Q^2}}\Phi^{(2)}_a(\xi,\zeta)\Big) \qquad (4.24)$$

derived in [20], cf. also [15, 16] for the case of forward scattering.

Most of the above quantities depend on $\mathcal{P}^2$ (3.15) which we write now as

$$\mathcal{P}^2 \;=\; p_+^2 + 2\zeta p_+\pi_- + \zeta^2\pi_-^2 = p_+^2(1+\epsilon)\ . \qquad (4.25)$$

Let us simplify the contraction of the kinematic coefficients in (4.17). For $\mathcal{K}_1 = p_+$, observing $q\mathcal{P} = qp_+$ and Eq. (4.25) for $\mathcal{P}^2$, we obtain

$$\frac{q\mathcal{K}_1}{q\mathcal{P}} = 1\,, \qquad (4.26)$$

$$\Big(\frac{q\mathcal{K}_1}{q\mathcal{P}} - \frac{\mathcal{P}\mathcal{K}_1}{\mathcal{P}^2}\Big)\frac{\mathcal{P}^2}{Q^2} = \frac{p_+^2}{Q^2}\Big(-\eta\zeta + \zeta^2\Big(\eta^2 + \frac{t}{p_+^2}\Big)\Big) \qquad (4.27)$$

$$\mathcal{K}^{\rm T}_{1\mu}\mathcal{P}^{\rm T}_\nu + \mathcal{P}^{\rm T}_\mu\mathcal{K}^{\rm T}_{1\nu} - 2\frac{q\mathcal{K}_1}{q\mathcal{P}}\mathcal{P}^{\rm T}_\mu\mathcal{P}^{\rm T}_\nu = -\zeta\left(p^{\rm T}_{+\mu}\pi_{-\nu} + p^{\rm T}_{+\nu}\pi_{-\mu}\right) - 2\,\zeta^2\pi_{-\mu}\pi_{-\nu}\,, \qquad (4.28)$$

and for $\mathcal{K}_2 = \pi_-$, due to the transversality of π_-, one finds

$$\frac{q\mathcal{K}_2}{q\mathcal{P}} = 0\,, \qquad (4.29)$$

$$\Big(\frac{q\mathcal{K}_2}{q\mathcal{P}} - \frac{\mathcal{P}\mathcal{K}_2}{\mathcal{P}^2}\Big)\frac{\mathcal{P}^2}{Q^2} = \frac{p_+^2}{Q^2}\Big(\eta - \zeta\Big(\eta^2 + \frac{t}{p_+^2}\Big)\Big), \qquad (4.30)$$

$$\mathcal{K}^{\rm T}_{2\mu}\mathcal{P}^{\rm T}_\nu + \mathcal{P}^{\rm T}_\mu\mathcal{K}^{\rm T}_{2\nu} - 2\frac{q\mathcal{K}_2}{q\mathcal{P}}\mathcal{P}^{\rm T}_\mu\mathcal{P}^{\rm T}_\nu = \left(p^{\rm T}_{+\mu}\pi_{-\nu} + p^{\rm T}_{+\nu}\pi_{-\mu}\right) + 2\,\zeta\,\pi_{-\mu}\pi_{-\nu}\,. \qquad (4.31)$$

It is remarkable that only for $\mathcal{K}_1 = p_+$ the first invariant $q\mathcal{K}_1/q\mathcal{P}$ contributes to the zeroth power in ζ, whereas the other ones start at most with the first power. The contributions of invariants belonging to kinematic coefficients containing π_- are less important because this variable is transverse to q with $\pi_-q = 0$. The corresponding invariants

$$\pi_-^2 = t + \eta^2\,p_+^2,\ \ \pi_-p_+ = -\eta\,p_+^2,\ \ \pi_-p_- = t\ , \qquad (4.32)$$

are small compared to Q^2.

Having now expressed the ζ–dependence in all kinematic factors explicitly, we may perform the ζ–integral introducing nth moments :

$$W^{(n)\,\rm diff}_{a\,k}(x,\eta,t,p_+^2/Q^2) = \int d\zeta\,\zeta^n\,W^{\rm diff}_{a\,k}(x,\mathcal{P}^2/Q^2;\zeta)\ . \qquad (4.33)$$

The hadronic tensor reads

$$\begin{aligned}
&\frac{1}{2\pi}\mathsf{Im}\,T^{\rm tw2}_{\{\mu\nu\}}(q) = \\
&\quad - g^{\rm T}_{\mu\nu}\left\{W^{(0)\,\rm diff}_{1\,1} + \frac{p_+^2}{Q^2}\left[\eta\,(W^{(0)\,\rm diff}_{2\,3} - W^{(1)\rm diff}_{1\,3}) + \left(\eta^2 + \frac{t}{p_+^2}\right)\,(W^{(2)\rm diff}_{1\,3} - W^{(1)\rm diff}_{2\,3})\right]\right\} \\
&\quad + \frac{p^{\rm T}_{+\,\mu}p^{\rm T}_{+\,\nu}}{M^2}\left\{W^{(0)\,\rm diff}_{1\,2} + \frac{p_+^2}{Q^2}\left[\eta\,\left(W^{(0)\,\rm diff}_{2\,4} - W^{(1)\,\rm diff}_{1\,4}\right) + \left(\eta^2 + \frac{t}{p_+^2}\right)\,(W^{(2)\,\rm diff}_{1\,4} - W^{(1)\,\rm diff}_{2\,4})\right]\right\}
\end{aligned}$$

$$
\begin{aligned}
&+ \frac{p^{\mathrm{T}}_{+\mu}\pi_{-\nu} + p^{\mathrm{T}}_{+\nu}\pi_{-\mu}}{M^2} \Big\{ W^{(1)\,\mathrm{diff}}_{1\,2} + \frac{p_+^2}{Q^2}\Big[\eta(W^{(1)\,\mathrm{diff}}_{2\,4} - W^{(2)\,\mathrm{diff}}_{1\,4}) \\
&\qquad\qquad + \left(\frac{t}{p_+^2} + \eta^2\right)(W^{(3)\,\mathrm{diff}}_{1\,4} - W^{(2)\,\mathrm{diff}}_{2\,4})\Big] + W^{(0)\,\mathrm{diff}}_{2\,5} - W^{(1)\,\mathrm{diff}}_{1\,5}\Big\} \\
&+ \frac{\pi_{-\mu}\pi_{+\nu}}{M^2} \Big\{ W^{(2)\,\mathrm{diff}}_{1\,2} + 2W^{(1)\,\mathrm{diff}}_{2\,5} - 2W^{(2)\,\mathrm{diff}}_{1\,5} \\
&\qquad + \frac{p_+^2}{Q^2}\left[\eta\left(W^{(2)\,\mathrm{diff}}_{2\,4} - W^{(3)\,\mathrm{diff}}_{1\,4}\right) + \left(\eta^2 + \frac{t}{p_+^2}\right)\left(W^{(4)\,\mathrm{diff}}_{1\,4} - W^{(3)\,\mathrm{diff}}_{2\,4}\right)\right]\Big\} . \qquad (4.34)
\end{aligned}
$$

Here the momentum fraction argument of the structure functions W^{diff}_{ak} is the original Nachtmann variable (3.18), whereas for the functions $W^{(n)\,\mathrm{diff}}_{a\,k}(x, \eta, t, p_+^2/Q^2)$ it is x. These structure functions are in principle accessible experimentally, varying the external kinematic parameters x_{BJ}, Q^2, t and $x_{\mathbb{P}}, (\eta = \eta(x_{\mathbb{P}}))$.

Up to this point no approximations have been made. We would now like to discuss the above structure. Note that disregarding of π_- as transversal degree of freedom corresponds to the limit $\epsilon \to 0$, (4.25). However, ϵ, (4.15), is not necessarily a small quantity. The Taylor expansion in ϵ would retain the DIS-like target mass corrections and lead to the physically relevant power series in p_+^2/Q^2 of the denominators. Because of the smallness of ρ (4.14) and also $x^2 p_+^2/Q^2$ we use the latter as expansion parameter. Thereby the Nachtmann variable is substituted by x in lowest order, whereas by setting $\pi_- = 0$ we would retain an approximate Nachtmann variable,

$$
\xi_0 = 2x/(1 + \sqrt{1 + x^2 p_+^2/Q^2}). \qquad (4.35)
$$

For simplicity we proceed as follows:

- effective expansion w.r.t. the parameter p_+^2/Q^2,

$$
\left(1 + x^2\,\frac{\mathcal{P}^2}{Q^2}\right)^{-n} = \left(1 + x^2\,\frac{p_+^2(1+\epsilon)}{Q^2}\right)^{-n} = \left(1 - nx^2\,\frac{p_+^2}{Q^2}(1+\epsilon) + \ldots\right), \qquad (4.36)
$$

- expansion of the Nachtmann variable (3.18),

$$
\xi - x = -\frac{1}{4} x \frac{x^2 p_+^2}{Q^2}(1+\epsilon) + \ldots , \qquad (4.37)
$$

- use of x instead of the Nachtmann variable ξ.

- For the treatment of the denominators we shift the integration variable $\lambda = \lambda' + \xi - x$,

$$
\frac{1}{\lambda - \xi + i\varepsilon\lambda} = \frac{1}{\lambda' - x + i\varepsilon\lambda}. \qquad (4.38)
$$

Through this procedure we avoid the expansion of the denominator in favor of an expansion of the dGPDs. In principle problems could arise because of possible differences in $\varepsilon(\lambda - \lambda')$. Therefore we have to expand the basic dGPD

$$
\begin{aligned}
\Phi_a^{(i)}(\lambda, \zeta) &= \Phi_a^{(i)}(\lambda' + \xi - x, \zeta) = \Phi_a^{(i)}(\lambda', \zeta) + \partial_{\lambda'}\Phi_a^{(i)}(\lambda', \zeta)(\xi - x) + \ldots \\
&= \Phi_a^{(i)}(\lambda', \zeta) - \frac{1}{4} x \frac{x^2 p_+^2}{Q^2}(1+\epsilon)\partial_{\lambda'}\Phi_a^{(i)}(\lambda', \zeta) + \ldots \quad .
\end{aligned}
$$

As a test we can study the limit of deep–inelastic scattering, whereby we reproduce the standard result. For diffractive DIS it is sufficient to consider the lowest approximation which extends our results [2,3]. In the following we define moments of the dGPDs by

$$\Phi_a^{(i\,n)}(x) = \int d\zeta \zeta^n \Phi_a^{(i)}(x,\zeta). \tag{4.39}$$

This corresponds to a change from a GPD to a parton density.

Now we apply our approximation procedure directly to Eq. (4.17) using (4.18–4.21) and (4.3–4.7). We write the result separately for the invariants $\mathcal{K}^1 = p_+$,

$$\begin{aligned}
\frac{1}{2\pi}\mathsf{Im}\, T^{\rm tw2}_{\{\mu\nu\}}(q)\,|_1 = g^{\rm T}_{\mu\nu}&\left[x\Phi_1^{(0\,0)}(x) + \frac{x^2p_+^2}{Q^2}\Big(t^0_{1\,1} + t^0_{1\,3} - \tilde{t}^0_{1\,3} + \eta\tilde{t}^1_{1\,3}\Big)\right]\\
&- \frac{p^{\rm T}_{+\,\mu}p^{\rm T}_{+\,\nu}}{Q^2}\left[x^3\Phi_1^{(0\,0)} + \frac{x^2p_+^2}{Q^2}x^2\Big(t^0_{1\,2} + t^0_{1\,4} - \tilde{t}^0_{1\,4} + \eta\tilde{t}^1_{1\,4}\Big)\right]\\
&- \frac{p^{\rm T}_{+\,\mu}\pi_{-\,\nu} + p^{\rm T}_{+\,\nu}\pi_{-\,\mu}}{Q^2}\Bigg[x^3\Phi_1^{(0\,1)} - x^2\Phi_1^{(1\,1)}\\
&\qquad\qquad + \frac{x^2p_+^2}{Q^2}x^2\Big(t^1_{1\,2} + t^1_{1\,4} - \tilde{t}^1_{1\,4} - t^1_{1\,5} + \eta\tilde{t}^2_{1\,4}\Big)\Bigg]\\
&- \frac{\pi_{-\,\mu}\pi_{-\,\nu}}{Q^2}\left[x^3\Phi_1^{(0\,2)} - 2x^2\Phi_1^{(1\,2)} + \frac{x^2p_+^2}{Q^2}x^2\Big(t^2_{1\,2} + t^2_{1\,4} - \tilde{t}^2_{1\,4} - 2t^2_{1\,5} + \eta\tilde{t}^3_{1\,4}\Big)\right],
\end{aligned} \tag{4.40}$$

and for $\mathcal{K}^2 = \pi_-$,

$$\begin{aligned}
\frac{1}{2\pi}\mathsf{Im}\, T^{\rm tw2}_{\{\mu\nu\}}(q)\,|_2 = g^{\rm T}_{\mu\nu}&\,\frac{x^2p_+^2}{Q^2}\left[\eta\tilde{t}^0_{2\,3} - \left(\eta^2 + \frac{t}{p_+^2}\right)\tilde{t}^1_{2\,3}\right]\\
&- \frac{p^{\rm T}_{+\,\mu}p^{\rm T}_{+\,\nu}}{Q^2}\frac{x^2p_+^2}{Q^2}x^2\left[\eta\tilde{t}^0_{2\,4} - \left(\eta^2 + \frac{t}{p_+^2}\right)\tilde{t}^1_{2\,4}\Big)\right]\\
&- \frac{p^{\rm T}_{+\,\mu}\pi_{-\,\nu} + p^{\rm T}_{+\,\nu}\pi_{-\,\mu}}{Q^2}\left[x^2\Phi_2^{(1\,0)} + \frac{x^2p_+^2}{Q^2}x^2\Big(t^0_{2\,5} + \eta t^1_{2\,4} - \left(\eta^2 + \frac{t}{p_+^2}\right)\tilde{t}^2_{2\,4}\Big)\right]\\
&- \frac{\pi_{-\,\mu}\pi_{-\,\nu}}{Q^2}\left[2x^2\Phi_2^{(1\,1)} + \frac{x^2p_+^2}{Q^2}x^2\Big(2t^1_{2\,5} + \eta t^2_{2\,4} - \left(\eta^2 + \frac{t}{p_+^2}\right)\tilde{t}^3_{2\,4}\Big)\right].
\end{aligned} \tag{4.41}$$

Here t^n_{ai} and $\tilde{t}^n_{ai}$ are given by

$$\begin{aligned}
t^n_{a1} &= \int d\zeta(1+\epsilon(\zeta))\zeta^n\left(-\tfrac{1}{2}x\Phi_a^{(0)}(x,\zeta) + \tfrac{1}{2}\Phi_a^{(1)}(x,\zeta) - \frac{1}{4}x^2\partial_x\Phi_a^{(0)}(x,\zeta)\right),\\
t^n_{a2} &= \int d\zeta(1+\epsilon(\zeta))\zeta^n\left(-\frac{3}{2}x\Phi_a^{(0)}(x,\zeta) + \frac{3}{2}\Phi_a^{(1)}(\xi_{00},\zeta) - \frac{1}{4}x^2\partial_x\Phi_a^{(0}(x,\zeta)\right),\\
t^n_{a3} &= \int d\zeta(1+\epsilon(\zeta))\zeta^n\left(-\tfrac{1}{2}x\Phi_a^{(0)}(x,\zeta) + \frac{1}{2}\Phi_a^{(1)}(\xi_{00},\zeta) - \int_x^1\frac{dy}{y}\Phi_a^{(1)}(y,\zeta)\right),
\end{aligned} \tag{4.42}$$

$$t^n_{a4} = \int d\zeta (1+\epsilon(\zeta))\zeta^n \left(-\tfrac{1}{2} x \Phi^{(0)}_a(x,\zeta) + \frac{5}{2}\Phi^{(1)}_a(x,\zeta) - 3\int_x^1 \frac{dy}{y}\Phi^{(1)}_a(y,\zeta) \right),$$

$$t^n_{a5} = \int d\zeta (1+\epsilon(\zeta))\zeta^n \left(-\frac{5}{4}\Phi^{(0)}_a(x,\zeta) + \frac{3}{2}\int_x^1 \frac{dy}{y}\Phi^{(1)}_a(y,\zeta) - \frac{1}{4} x\partial_x \Phi^{(1)}_a(x,\zeta) \right), \tag{4.43}$$

and

$$\tilde{t}^n_{a1} = \int d\zeta \zeta^n \left(-\tfrac{1}{2} x \Phi^{(0)}_a(x,\zeta) + \tfrac{1}{2}\Phi^{(1)}_a(x,\zeta) - \frac{1}{4} x^2 \partial_x \Phi^{(0)}_a(x,\zeta) \right) . \tag{4.44}$$

Similar for all other terms $\tilde{t}^n_{ak}$, the factor $(1+\epsilon(\zeta))$ is absent compared to t^n_{ak}.

It is remarkable that each kinematic coefficient $\mathcal{K}^a$ contributes to all possible kinematic structures. Because of the transversal behaviour of π_- we expect that the last two structures $p^{\rm T}_{+\,\mu}\pi_{-\,\nu} + p^{\rm T}_{+\,\nu}\pi_{-\,\mu}$ and $\pi_{-\,\mu}\pi_{-\,\nu}$ as well as the complete contributions of the second invariant (4.41) are less important in comparison with the structures $g^{\rm T}_{\mu\nu}$ and $p^{\rm T}_{+\,\mu}p^{\rm T}_{+\,\nu}$ and the first invariant in (4.40). Moreover the leading contributions to the first two structures in (4.41) contain the small coefficient η. In Eqs. (4.40, 4.41) the contributions $\propto M^2, t$ emerge as

$$\frac{x^2 p_+^2}{Q^2} = \frac{x^2\,(4M^2 - t)}{Q^2} , \tag{4.45}$$

$$\frac{x^2 p_-^2}{Q^2} = \frac{x^2 t}{Q^2} , \tag{4.46}$$

respectively. Noting that $|\eta| \simeq x_{\mathbb{P}}$, and $x_{\mathbb{P}} \sim \mathcal{O}(x_{\rm BJ})$ for diffractive scattering the target mass and finite t corrections are suppressed by $\mathcal{O}(x^2 M^2/Q^2)$, with $x \lesssim 10^{-2}$. In the meson-exchange case, x–values of around $x \simeq 0.3$ may be reached and $\mathcal{O}(10\% \times (M^2/Q^2))$ effects may be obtained.

Let us consider the complete zeroth order term

$$\begin{aligned} \frac{1}{2\pi}\mathsf{Im}\, T^{\rm tw2}_{\{\mu\nu\}}(q)\,|_0 = & g^{\rm T}_{\mu\nu}\; x\Phi^{(0\,0)}_1(x) - \frac{p^{\rm T}_{+\,\mu}p^{\rm T}_{+\,\nu}}{Q^2} x^3\,\Phi^{(0\,0)}_1 \\ & - \frac{p^{\rm T}_{+\,\mu}\pi_{-\,\nu} + p^{\rm T}_{+\,\nu}\pi_{-\,\mu}}{Q^2} x^2 \left[x\,\Phi^{(0\,1)}_1 + \Phi^{(1\,0)}_2 - \Phi^{(1\,1)}_1 \right] \\ & - \frac{\pi_{-\,\mu}\pi_{-\,\nu}}{Q^2}\, x^2 \left[x\,\Phi^{(0\,2)}_1 - 2\Phi^{(1\,2)}_1 + 2\Phi^{(1\,1)}_2 \right] . \end{aligned} \tag{4.47}$$

Also here we can see that the contributions to the first two kinematic structures result from the distribution functions $\Phi^{(0\,0)}_1(x)$ of the first kinematic structure only. This reproduces our result [2] obtained for vanishing t, target mass, and negligible transversal momenta π_-,

$$\frac{1}{2\pi}\mathsf{Im}\, T^{\rm tw2}_{\{\mu\nu\}}(q)\,|_0 = g^{\rm T}_{\mu\nu}\; x\Phi^{(0\,0)}_1(x) - \frac{p^{\rm T}_{+\,\mu}p^{\rm T}_{+\,\nu}}{Q^2} x^3\,\Phi^{(0\,0)}_1 .$$

The leading t-dependence is contained in the first structure $g^{\rm T}_{\mu\nu}$ of (4.40)

$$\frac{1}{2\pi}\mathsf{Im}\, T^{\rm tw2}_{\{\mu\nu\}}(q)\,|_t = g^{\rm T}_{\mu\nu}\,\frac{x^2 t}{Q^2}\,\chi(x), \tag{4.48}$$

$$\begin{aligned} \chi(x) \approx & \Big\{ \frac{1}{2} x \big(\Phi^{(0\,0)}_1 - \Phi^{(0\,2)}_1 - \eta(3\Phi^{(0\,1)}_1 + \Phi^{(0\,3)}_1) \big) - \frac{1}{2} \big(\Phi^{(1\,0)}_1 - \Phi^{(1\,2)}_1 - \eta(3\Phi^{(1\,1)}_1 + \Phi^{(1\,3)}_1) \big) \\ & + \frac{1}{4} x^2 \partial_x (\Phi^{(0\,0)}_1 - \Phi^{(0\,2)}_1 - 2\eta\Phi^{(0\,1)}_1) - \int_x^1 \frac{dy}{y} (\eta\Phi^{(1\,1)}_1 + \eta\Phi^{(1\,3)}_1 + \Phi^{(1\,2)}_1) \Big\} . \end{aligned} \tag{4.49}$$

Terms $\propto \eta^2 \simeq x_{\mathbb{P}}^2$ are dropped.

A last remark concerns the generalized Callan-Gross relation (4.22). This relation can be written for ζ–moments (4.33) as

$$\begin{aligned} W_{a\,\mathrm{L}}^{(n)\mathrm{diff}}(x, p_+^2/Q^2) &= -W_{a\,1}^{(n)\mathrm{diff}}(x, p_+^2/Q^2) + \frac{p_+^2 + (qp_+)^2/Q^2}{M^2} W_{a\,2}^{(n)\mathrm{diff}}(x, p_+^2/Q^2) \\ &+ \frac{p_+^2}{M^2} \int d\zeta \zeta^n \, \epsilon \, W_{a\,2}^{\mathrm{diff}}(x, \mathcal{P}^2/Q^2, \zeta). \end{aligned} \tag{4.50}$$

Finally we remark that an equivalent kinematic parameterization can be obtained using

$$p_\pm^{\mathrm{T}} = p_2^{\mathrm{T}} \pm p_1^{\mathrm{T}}, \qquad \pi_- = p_- - \eta p_+ = p_2(1-\eta) - p_1(1+\eta) = p_2^{\mathrm{T}}(1-\eta) - p_1^{\mathrm{T}}(1+\eta). \tag{4.51}$$

These relations allow to link different representations of the hadronic tensor, which linearly relates various definitions of structure functions, cf. (2.13). All contributions due to M^2 and t–effects in the above are suppressed like $\propto x_{(\mathbb{P})}^2 \mu^2/Q^2$ with $\mu^2 = |t|, M^2$.

There are, however also other contributions emerging in the scattering cross sections, which are of kinematic origin and stem from 4-vector products contributing to the process contracting the leptonic and hadronic tensor, see Appendix A for details. Most of these invariants are large, like $l_1.l_2$ and $l_1.p_1$. The invariant $l_1.p_2$, (A.37), leads to kinematic power corrections further to those considered in (2.24–2.26). Here the leading contribution beyond the lowest order term is of $\mathcal{O}(\cos(\phi_b) x_{\mathrm{BJ}} \sqrt{\mu^2/Q^2}), \mu^2 = |t|, M^2$. The terms, which do not vary with the angle ϕ_b are of $\mathcal{O}(x_{\mathrm{BJ}} \mu^2/Q^2)$. In conclusion, the largest dependences from the limiting case $|t|, M^2 \to 0$ are obtained from the kinematic terms in the cross section. Those resulting from the target-mass and t-corrections of the hadronic matrix elements always occur with an extra power in x_{BJ} or $x_{\mathbb{P}}$.

4.2 The Antisymmetric Part

The contribution to the antisymmetric part of the hadronic tensor is given by, cf. [1,20],

$$\begin{aligned} W_{[\mu\nu]}^{\mathrm{tw2}}(q) = -\pi \, \epsilon_{\mu\nu}{}^{\alpha\beta} \int d\zeta \Bigg\{ & \frac{q_\alpha \, \mathcal{K}_{5\beta}^a}{q\mathcal{P}} \Big[g_{a1}(x;\zeta) + g_{a2}(x;\zeta) \Big] - \frac{q_\alpha \, \mathcal{P}_\beta}{q\mathcal{P}} \frac{(q\mathcal{K}_5^a)}{q\mathcal{P}} \, g_{a2}(x;\zeta) \\ & + \frac{1}{2} \frac{q_\alpha \, \mathcal{P}_\beta}{q\mathcal{P}} \frac{(\mathcal{P}\mathcal{K}_5^a)}{Q^2} \, g_{a0}(x;\zeta) \Bigg\}, \end{aligned} \tag{4.52}$$

in terms of the ζ-integral. Here the coefficients $\mathcal{K}_{5\,\mu}^a$ are given by (3.8) and the functions $g_{a\,k}(x;\zeta) \equiv g_{a\,k}(x, \xi, \mathcal{P}^2/Q^2, \zeta)|_{k=0,1,2}$ read

$$g_{a1}(x;\zeta) = \frac{x}{\xi} \frac{1}{[1 + x^2 \, \mathcal{P}^2/Q^2]^{3/2}} \times \tag{4.53}$$
$$\left[\Phi_{5a}^{(0)}(\xi,\zeta) + \frac{x(\xi + x)\, \mathcal{P}^2/Q^2}{[1 + x^2\mathcal{P}^2/Q^2]^{1/2}} \Phi_{5a}^{(1)}(\xi,\zeta) - \frac{x\xi \, \mathcal{P}^2}{2\,Q^2} \frac{2 - x^2\mathcal{P}^2/Q^2}{1 + x^2\mathcal{P}^2/Q^2} \Phi_{5a}^{(2)}(\xi,\zeta) \right],$$

$$g_{a2}(x;\zeta) = -\frac{x}{\xi} \frac{1}{[1 + x^2\mathcal{P}^2/Q^2]^{3/2}} \times \tag{4.54}$$
$$\left[\Phi_{5a}^{(0)}(\xi,\zeta) - \frac{1 - x\xi \, \mathcal{P}^2/Q^2}{[1 + x^2\mathcal{P}^2/Q^2]^{1/2}} \Phi_{5a}^{(1)}(\xi,\zeta) - \frac{3}{2} \frac{x\xi \, \mathcal{P}^2/Q^2}{1 + x^2\mathcal{P}^2/Q^2} \Phi_{5a}^{(2)}(\xi,\zeta) \right],$$

$$
\begin{aligned}
& g_{a1}(x;\zeta) + g_{a2}(x;\zeta) \\
&\quad = \frac{x}{\xi}\frac{1}{[1+x^2\mathcal{P}^2/Q^2]^{3/2}}\left[\left(1+\frac{x\xi\mathcal{P}^2}{2Q^2}\right)\Phi^{(1)}_{5a}(\xi,\zeta)\,+\frac{x\xi\mathcal{P}^2}{2Q^2}\Phi^{(2)}_{5a}(\xi,\zeta)\right] \qquad (4.55)\\
&\quad = \frac{x}{\xi}\frac{1}{1+x^2\mathcal{P}^2/Q^2}\left[\Phi^{(1)}_{5a}(\xi,\zeta)\,+\frac{1}{2}\,\frac{x\xi\,\mathcal{P}^2/Q^2}{[1+x^2\mathcal{P}^2/Q^2]^{1/2}}\Phi^{(2)}_{5a}(\xi,\zeta)\right],\\
& g_{a0}(x;\zeta) = \frac{x^2}{[1+x^2\mathcal{P}^2/Q^2]^{3/2}}\times \qquad (4.56)\\
&\qquad \left[\Phi^{(0)}_{5a}(\xi,\zeta) - \frac{3}{[1+x^2\mathcal{P}^2/Q^2]^{1/2}}\Phi^{(1)}_{5a}(\xi,\zeta) + \frac{2-x^2\mathcal{P}^2/Q^2}{1+x^2\mathcal{P}^2/Q^2}\Phi^{(2)}_{5a}(\xi,\zeta)\right].
\end{aligned}
$$

The dGPDs $\Phi^{(i)}_{5\,a}(\xi,\zeta)$ are based on (3.6) and, similar to the definitions (4.8–4.11) of $\Phi^{(i)}_{a}(\xi,\zeta)$,

$$
\Phi^{(0)}_{5\,a}(\xi,\zeta) \;\equiv\; \xi\, f_{5\,a}(\xi,\zeta)\,, \qquad (4.57)
$$

$$
\Phi^{(i)}_{5\,a}(\xi,\zeta) \;=\; \int_\xi^1 \frac{dy}{y}\,\Phi^{(i-1)}_{5\,a}(y,\zeta)\,, i\geq 1\,. \qquad (4.58)
$$

As shown before [1, 20] the Wandzura–Wilczek (WW) relation [35] holds for the un-integrated distribution functions

$$
g^{\rm tw2}_{a2}(x;\zeta) \;=\; -\,g^{\rm tw2}_{a1}(x;\zeta) + \int_x^1 \frac{dy}{y} g^{\rm tw2}_{a1}(y;\zeta)\,, \qquad (4.59)
$$

between $g^{\rm tw2}_{a2}$ and $g^{\rm tw2}_{a1}$. All target mass and t–corrections can uniquely be absorbed into the structure functions. Note that this relation holds for all invariants $\mathcal{K}^a_5$ independently. The validity of the Wandzura–Wilczek relation for diffractive scattering at general hadronic scales M^2, t is a further example in a long list of cases. It was observed using the covariant parton model and light-cone expansion [36, 37]. For forward scattering, target- and quark-mass corrections could be completely absorbed into the structure functions maintaining the WW-relation [16, 17]. It is valid for gluon-induced heavy flavor production [38], non–forward scattering [39], and diffractive scattering in the limit $M^2, t \to 0$ [3]. In the electro–weak case further sum–rules exist [37]. Considering the target mass corrections there are new twist–3 integral relations [16]. The distribution function $g^{\rm tw2}_{a0}$ is also related to $g^{\rm tw2}_{a1}$ but in a more complicated manner:

$$
\begin{aligned}
g^{\rm tw2}_{a0}(x;\zeta) \;=\;& x\xi\, g^{\rm tw2}_{a1}(x;\zeta) - \frac{2x^2+x\xi}{[1+x^2\mathcal{P}^2/Q^2]^{1/2}}\int_x^1\frac{dy}{y}g^{\rm tw2}_{a1}(y;\zeta)\\
&+\frac{2x^2}{[1+x^2\mathcal{P}^2/Q^2]^{3/2}}\int_x^1\frac{dy}{y}\int_y^1\frac{dy'}{y'}g^{\rm tw2}_{a1}(y';\zeta)\,. \qquad (4.60)
\end{aligned}
$$

From (4.52) – (4.56) we now extract the ζ-independent functions. In the kinematic factors ζ appears only up to second power. As preliminary classification we therefore can perform the ζ–integrals according to the ζ–powers of the kinematic factors, not counting the internal ζ-dependence of the GPDs $g_{ak}(x;\zeta)$ itself.

For each invariant a let us define

$$
G^{(n)}_{a\,k}(x,\eta,t,p_+^2/Q^2) = \int d\zeta\,\zeta^n\, g_{a\,k}(x;\zeta)\,, \qquad k = 0,1,2, \qquad (4.61)
$$

so that from (4.52) one obtains

$$
\begin{aligned}
\mathrm{Im}\, T^{\mathrm{tw2}}_{[\mu\nu]}(q) = -\pi\, \epsilon_{\mu\nu}{}^{\alpha\beta} \Bigg\{ & \frac{q_\alpha\, \mathcal{K}^a_{5\beta}}{qp_+} \Big(G^{(0)}_{a1} + G^{(0)}_{a2} \Big) \\
& - \frac{q_\alpha\, p_{+\beta}}{qp_+} \left(\frac{q\mathcal{K}^a_5}{qp_+} G^{(0)}_{a2} - \frac{1}{2} \frac{p_+\mathcal{K}^a_5}{Q^2} G^{(0)}_{a0} - \frac{1}{2} \frac{\pi_-\mathcal{K}^a_5}{Q^2} G^{(1)}_{a0} \right) \\
& - \frac{q_\alpha\, \pi_{-\beta}}{qp_+} \left(\frac{q\mathcal{K}^a_5}{qp_+} G^{(1)}_{a2} - \frac{1}{2} \frac{p_+\mathcal{K}^a_5}{Q^2} G^{(1)}_{a0} - \frac{1}{2} \frac{\pi_-\mathcal{K}^a_5}{Q^2} G^{(2)}_{a0} \right) \Bigg\} . \qquad (4.62) \\
\approx -\pi\, \epsilon_{\mu\nu}{}^{\alpha\beta} \Bigg\{ & \frac{q_\alpha\, \mathcal{K}^a_{5\beta}}{qp_+} \Big(G^{(0)}_{a1} + G^{(0)}_{a2} \Big) - \frac{q_\alpha\, p_{+\beta}}{qp_+} \frac{q\mathcal{K}^a_5}{qp_+} G^{(0)}_{a2} - \frac{q_\alpha\, \pi_{-\beta}}{qp_+} \frac{q\mathcal{K}^a_5}{qp_+} G^{(1)}_{a2} \Bigg\} . \qquad (4.63)
\end{aligned}
$$

In the last line the leading terms are written only. Now, inserting the three invariants $\mathcal{K}^a_5$ in (3.8) explicitly, one obtains disregarding sub-leading terms in $1/Q^2$:

$$
\begin{aligned}
\mathrm{Im}\, T^{(0)\,\mathrm{tw2}}_{[\mu\nu]}(q) \approx -\pi\, \epsilon_{\mu\nu}{}^{\alpha\beta} \Bigg\{ & \frac{q_\alpha S^{\mathrm{T}}_\beta}{qp_+} \big(G^{(0)}_{1\,1} + G^{(0)}_{1\,2} \big) - \frac{q_\alpha p^{\mathrm{T}}_{+\beta}}{qp_+} \left[\frac{qS}{qp_+} G^{(0)}_{1\,2} - \frac{p_2.S}{M^2} G^{(0)}_{2\,1} \right] \\
& - \frac{q_\alpha \pi^{\mathrm{T}}_{-\beta}}{qp_+} \left[\frac{qS}{qp_+} G^{(1)}_{1\,2} + \frac{p_2.S}{M^2} \left(G^{(1)}_{2\,2} - G^{(0)}_{3\,1} - G^{(0)}_{3\,2} \right) \right] \Bigg\} . \qquad (4.64)
\end{aligned}
$$

Due to the presence of the Levi-Civita symbol, only transversal components (2.17) contribute. Note that $p_1.S = 0$. The approximate expressions for $G^{(n)}_{a\,k}$ are

$$
\begin{aligned}
G^{(n)}_{a1}(x) &\approx \Phi^{(0\,n)}_{5a}(x) - \frac{x^2\, p_+^2}{Q^2} \gamma^n_{a1} \\
G^{(n)}_{a2}(x) &\approx - \left[\Phi^{(0\,n)}_{5a}(x) - \Phi^{(1\,n)}_{5a}(x) \right] - \frac{x^2\, p_+^2}{Q^2} \gamma^n_{a2}, \\
G^{(n)}_{a1}(x) + G^{(n)}_{a2}(x) &\approx \Phi^{(1\,n)}_{5a}(x) \ - \frac{x^2\, p_+^2}{Q^2} \gamma^n_{a12} \\
G^{(n)}_{a0}(x) &\approx x^2 \left[\Phi^{(0\,n)}_{5a}(x) - 3\Phi^{(1\,n)}_{5a}(x) + 2\Phi^{(2\,n)}_{5a}(x) \right] - \frac{x^2\, p_+^2}{Q^2} \gamma^n_{a0} \ , \qquad (4.65)
\end{aligned}
$$

with

$$
\begin{aligned}
\gamma^n_{a1} &= \int d\zeta (1 + \epsilon(\zeta)) \zeta^n \left(\frac{5}{4} \Phi^{(0)}_{a5}(x,\zeta) - 2\Phi^{(1)}_{a5}(x,\zeta) + \Phi^{(2)}_{a5}(x,\zeta) + \frac{1}{4} x \partial_x \Phi^{(0)}_{a5}(x,\zeta) \right), \\
\gamma^n_{a2} &= \int d\zeta (1 + \epsilon(\zeta)) \zeta^n \left(-\frac{5}{4} \Phi^{(0)}_{a5}(x,\zeta) + \frac{11}{4} \Phi^{(1)}_{a5}(x,\zeta) - \frac{3}{2} \Phi^{(2)}_{a5}(x,\zeta) \right. \\
& \qquad \left. - \frac{1}{4} x \partial_x \big(\Phi^{(0}_{a5}(x,\zeta) - \Phi^{(1)}_{a5}(x,\zeta) \big) \right), \\
\gamma^n_{a12} &= \int d\zeta (1 + \epsilon(\zeta)) \zeta^n \left(-\tfrac{1}{2} x \Phi^{(2)}_{a5}(x,\zeta) + \frac{3}{4} \Phi^{(1)}_{a5}(x,\zeta) + \frac{1}{4} x \partial_x \Phi^{(1)}_{a5}(x,\zeta) \right)
\end{aligned}
$$

and

$$
\widetilde{\gamma}^n_{a1} = \int d\zeta \zeta^n \left(\frac{5}{4} \Phi^{(0)}_{a5}(x,\zeta) - 2\Phi^{(1)}_{a5}(x,\zeta) + \Phi^{(2)}_{a5}(x,\zeta) + \frac{1}{4} x \partial_x \Phi^{(0)}_{a5}(x,\zeta) \right) \ ,
$$

with similar expressions for the other terms $\widetilde{\gamma}^n_{ak}$. The functions $\Phi^{(1\,n)}_{5a}(x)$ are determined as in Eq. (4.39). The last function $G^{(n)}_{a0}$ does not contribute to leading order. As final approximate result in leading order we obtain

$$
\begin{aligned}
\mathrm{Im}\, T^{(0)\,\mathrm{tw2}}_{[\mu\nu]}(q)\,|_0 \approx -\pi\,\epsilon_{\mu\nu}{}^{\alpha\beta} \Bigg\{ & \frac{q_\alpha S^{\mathrm{T}}_\beta}{qp_+}\Phi^{(1,0)}_{51}(x) \\
& + \frac{q_\alpha p^{\mathrm{T}}_{+\beta}}{qp_+}\left[\frac{qS}{qp_+}\left(\Phi^{(0,0)}_{51}(x)-\Phi^{(1,0)}_{51}(x)\right)+\frac{p_2 S}{M^2}\Phi^{(0,0)}_{52}(x)\right] \\
& + \frac{q_\alpha \pi^{\mathrm{T}}_{-\beta}}{qp_+}\left[\frac{qS}{qp_+}\left(\Phi^{(0,1)}_{51}(x)-\Phi^{(1,1)}_{51}(x)\right)+\frac{p_2 S}{M^2}\left(\Phi^{(1,0)}_{53}(x)+\Phi^{(0,1)}_{52}(x)-\Phi^{(1,1)}_{52}(x)\right)\right]\Bigg\}\,.
\end{aligned}
\tag{4.66}
$$

In Ref. [3] the terms $\propto p_2.S$ were neglected treating $p_2||p_1$ and vanishing contributions $\propto \pi_-$. While this is correct for $t\to 0$, a finite contribution remains for $M^2\to 0$, Eq. (A.23),

$$
\lim_{t\to 0}\frac{p_2.S}{M^2} = x_{\mathbb{P}}\frac{1-x_{\mathbb{P}}/2}{1-x_{\mathbb{P}}} + \mathcal{O}\left(x_{\mathrm{BJ}}x_{\mathbb{P}}\frac{M^2}{Q^2}\right)\,. \tag{4.67}
$$

So the previous result [3] is to be modified by a third term

$$
\mathrm{Im}\, T^{(0)\,\mathrm{tw2}}_{[\mu\nu]}(q)\,|_0 \approx -\pi\,\epsilon_{\mu\nu}{}^{\alpha\beta}\left\{\frac{q_\alpha S^{\mathrm{T}}_\beta}{qp_+}\Phi^{(1,0)}_{51}(x)+\frac{q_\alpha p^{\mathrm{T}}_{+\beta}}{qp_+}\left[\frac{qS}{qp_+}\left(\Phi^{(0,0)}_{51}(x)-\Phi^{(1,0)}_{51}(x)\right)+x_{\mathbb{P}}\Phi^{(0,0)}_{52}(x)\right]\right\}\,. \tag{4.68}
$$

The t-dependent correction terms result from the (η^2+t/p_+^2)–contributions in ϵ and they are entirely contained in correction terms γ^n_{ak} :

$$
\begin{aligned}
\mathsf{Im}\, T^{(0)\,\mathrm{tw2}}_{[\mu\nu]}(q)\,|_t \;\approx\; +\pi\,\epsilon_{\mu\nu}{}^{\alpha\beta}\frac{x^2\,t}{Q^2}\Bigg\{ & \frac{q_\alpha S^{\mathrm{T}}_\beta}{qp_+}\left((\widetilde{\gamma}^2_{1\,1}+\widetilde{\gamma}^2_{1\,2})+2\eta(\widetilde{\gamma}^1_{1\,1}+\widetilde{\gamma}^1_{1\,2})-(\widetilde{\gamma}^0_{1\,1}+\widetilde{\gamma}^0_{1\,2})\right) \\
& -\frac{q_\alpha p^{\mathrm{T}}_{+\beta}}{qp_+}\left[\frac{qS}{qp_+}\left(\widetilde{\gamma}^2_{1\,2}+2\eta\widetilde{\gamma}^1_{1\,2}-\widetilde{\gamma}^0_{1\,2}\right)-\frac{p_2 S}{M^2}\left(\widetilde{\gamma}^2_{2\,1}+2\eta\widetilde{\gamma}^1_{2\,1}-\widetilde{\gamma}^0_{2\,1}\right)\right] \\
& -\frac{q_\alpha \pi^{\mathrm{T}}_{-\beta}}{qp_+}\Bigg[\frac{qS}{qp_+}\left(\widetilde{\gamma}^3_{1\,2}+2\eta\widetilde{\gamma}^2_{1\,2}-\widetilde{\gamma}^0_{1\,2}\right) \\
& \quad +\frac{p_2 S}{M^2}\Big((\widetilde{\gamma}^3_{2\,2}-\widetilde{\gamma}^2_{3\,1}-\widetilde{\gamma}^2_{3\,2})+2\eta(\widetilde{\gamma}^2_{2\,2}-\widetilde{\gamma}^1_{3\,1}-\widetilde{\gamma}^1_{3\,2}) \\
& \qquad -(\widetilde{\gamma}^1_{2\,2}-\widetilde{\gamma}^0_{3\,1}-\widetilde{\gamma}^0_{3\,2})\Big)\Bigg]\Bigg\}\,.
\end{aligned}
\tag{4.69}
$$

The corresponding terms are of the same size as in the unpolarized case, Section 4.1, and may have a quantitative effect only in the low Q^2–region in the meson–exchange case.

It is remarkable that the Wandzura-Wilczek relation [35] remains intact after ζ-integrations and is valid for the experimentally observable moments,

$$
G^{(n)}_{a\,2}(x,\eta,t,p_+^2/Q^2) = -\,G^{(n)}_{a\,1}(x,\eta,t,p_+^2/Q^2) + \int_x^1\frac{dy}{y}\,G^{(n)}_{a\,1}(y,\eta,t,p_+^2/Q^2)\,. \tag{4.70}
$$

The second integral relation (4.60) contains the ζ–dependent denominator $\sqrt{1+x^2\mathcal{P}^2/q^2}$ so that we obtain after ζ-integration more complicated expressions. In the approximation $\pi_-=0$ one

obtains

$$G^{(n)}_{a\,0}(x,\eta,t,p_+^2/Q^2) \approx x\xi_0\, G^{(n)}_{a\,1}(x,\eta,t,p_+^2/Q^2) - \frac{2x^2+x\xi_0}{[1+4x^2\,p_+^2/Q^2]^{1/2}} \int_x^1 \frac{dy}{y}\, G^{(n)}_{a\,1}(y,\eta,t,p_+^2/Q^2)$$
$$+ \frac{2x^2}{[1+4x^2\,p_+^2/Q^2]^{3/2}} \int_x^1 \frac{dy}{y} \int_y^1 \frac{dy'}{y'}\, G^{(n)}_{a\,1}(y',x,\eta,t,p_+^2/Q^2)\,. \qquad (4.71)$$

ξ_0 denotes the Nachtmann variable (4.35). However the functions $G^{(n)}_{a\,0}(x,\eta,t,p_+^2/Q^2)$ contribute to sub-leading terms only.

5 Conclusions

Deep–inelastic diffractive scattering, like other hard scattering processes off nucleons, requires target mass corrections in the region of lower Q^2-scales. In fact, the nucleon mass M is not the only hadronic scale relevant to that process where both the incoming and outgoing nucleon play a role. The invariant $t=(p_2-p_1)^2$ on average is of the same size as M^2.[3] In the present paper we investigated in detail the conditions under which terms like M^2/Q^2 or $|t|/Q^2$ contribute.

We considered the leading twist contributions for which factorization theorems allow a partonic description. With the help of A. Mueller's generalized optical theorem it was possible to reformulate diffractive scattering in terms of deep–inelastic scattering off an effective two–nucleon pseudo-state accounting for t. All essential expressions determining experimentally relevant quantities are the diffractive generalized parton densities (dGPD) defined as expectation values of non-local light-cone operators (3.5,3.6). The involved iterated diffractive dGPDs (4.8–4.13), respectively (4.57) and (4.58), $\Phi^{(i)}_{(5)a}(\lambda,\zeta,t,\eta;\mu^2)$ depend on at least three variables, λ,ζ and t. Hereby t is an external variable, whereas λ is defined as overall scale multiplied with a generalized momentum in $(p_+z_+ + p_-z_-) = \lambda\mathcal{P}$. In the hadronic tensor $W_{\mu\nu} = \mathrm{Im}\, T_{\mu\nu}$ it is fixed by ξ, the generalized Nachtmann variable (3.18). Moreover the generalized momentum $\mathcal{P} = p_+ + \zeta\pi_-$ splits into a "longitudinal" and a "transversal" part π_- multiplied by a new variable ζ and can be treated separately. The problem in applying the results of our previous work [1] is the dependence of the dGPDs on the 'internal' variable ζ which is not measurable in experiment since it contributes through a definite integral in the final expressions. We performed an expansion w.r.t. the external variable p_+^2/Q^2. This leads to a set of integrated dGPDs which describe the process and the relevant mass corrections in a well-defined approximation.

One of our results is a prescription of experimental data in terms of experimentally accessible integrated diffractive GPD's,

$$\Phi^{(i\,n)}_{(5)a}(\xi,t,\eta) = \int d\zeta\, \zeta^n\, \Phi^{(i)}_{(5)a}(\xi,\zeta,t,\eta)\,, \qquad (5.1)$$

or approximately by the functions (4.39), which could be considered as diffractive parton densities, as it is the case for vanishing masses [2]. For our approximation a similar relation holds, where ξ is substituted by the variable $x=Q^2/qp_+$. Note that one and the same diffractive input GPD $\Phi^{(i)}_{(5)a}(\xi,\zeta,t,\eta)$ determines several amplitudes with different kinematic factors. This can be seen in the lowest approximations (4.47) or (4.66) and for the t-dependent corrections (4.49) and (4.69).

[3] In case of related semi–exclusive processes in which more than one final–state hadron is well separated in rapidity from the inclusively produced hadrons other invariants more would emerge.

The t– and M^2–dependence due to the functions $\Phi^{(in)}_{(5)a}(\xi, t, \eta)$, besides the non-perturbative t–behaviour, turns out to be of $\mathcal{O}(x^2_{\rm BJ(\mathbb{P})}\mu^2/Q^2)$, $\mu^2 = |t|, M^2$. Some of the kinematic factors emerging in the scattering cross section turn out to be less suppressed and are of $\mathcal{O}(x_{\rm BJ(\mathbb{P})}\mu^2/Q^2)$. In the case of diffractive scattering the region of $x_{\rm BJ}$ and $x_{\mathbb{P}}$ is effectively limited by $\lessapprox 10^{-2}$. The corresponding corrections cannot be resolved at the experimental accuracy. The effects are larger in the case of meson–exchange processes with a fast hadron due to the range $x \lessapprox 0.3$. Due to the smallness of these corrections the diffractive distribution functions obey a partonic description, where t plays the role of an additional variable besides $\beta = x_{\rm BJ}/x_{\mathbb{P}}$.

At the level of twist–2 the structure functions the scattering cross section can be built from the corresponding operator expectation values (3.5–3.6) as in the case of deep–inelastic scattering since the specifics of diffractive scattering is moved into the corresponding two–particle wave functions. Consequently, the logarithmic scaling violations, which can be completely associated with that of the operators, cf. [2,27], are found to be the same as in DIS or DVCS, if the complete diffractive GPDs are used.

The integral relations (4.22), (4.59) and (4.60) can be transformed in part to the integrated functions only. The presence of target mass and t-effects enlarges the number of structure functions determining the hadronic tensor if compared to the case of forward scattering. As shown in the present paper, these corrections are suppressed by at least one power in $x_{\rm BJ}$ or $x_{\mathbb{P}}$ and therefore the picture derived in [2, 3] remains valid quantitatively. In the polarized case, there is a new term, cf. (4.68), which contains $x_{\mathbb{P}}$ as prefactor. The Wandzura–Wilzcek relation remains unbroken and holds even separately for the contributions of the three different invariants $\mathcal{K}^5_a$ (3.8). We have also shown how the present formalism can be used to derive the target mass corrections in the limit of forward scattering.

A Kinematic Relations

In the following we list kinematic relations for the process of deeply-inelastic diffractive scattering. The incoming and outgoing lepton momenta are l_1 and l_2, those of the nucleon are p_1 and p_2 (diffractive nucleon), and the vector of the remainder hadrons is denoted by r. We disregard the lepton masses, $l_1.l_1 = l_2.l_2 = 0$. The kinematic invariants of this $2 \to 3$ particle scattering process are, cf. [40],

$$
\begin{aligned}
p_1.p_1 &= p_2.p_2 = M^2 \ , && \text{(A.1)}\\
r.r &= M_X^2 \ , && \text{(A.2)}\\
s &= (l_1 + p_1)^2 = 2l_1.p_1 + M^2 \ , && \text{(A.3)}\\
q^2 &= -Q^2 = (l_1 - l_2)^2 = -2l_1.l_2 \ , && \text{(A.4)}\\
t &= (p_1 - p_2)^2 = 2M^2 - 2p_1.p_2 \ , && \text{(A.5)}\\
W^2 &= (r + p_2)^2 = (q + p_1)^2 = Q^2\left(\frac{1}{x_{\rm BJ}} - 1\right) + M^2 \ , && \text{(A.6)}\\
l_1.q &= -Q^2/2 \ , && \text{(A.7)}\\
l_2.q &= +Q^2/2 \ , && \text{(A.8)}\\
s_1 &= (l_2 + r)^2 \ , && \text{(A.9)}\\
2l_1.p_2 &= s - s_1 + t - M^2 \ . && \text{(A.10)}
\end{aligned}
$$

For the later analysis it will be useful to consider the cms frame of the momenta

$$\boldsymbol{p}_1 + \boldsymbol{q} = \boldsymbol{p}_2 + \boldsymbol{r} = 0 \ . \qquad \text{(A.11)}$$

We need to express $S_{||}.p_2$. This requires a suitable representation of p_2, which cannot be obtained from the invariants above. In the frame (A.11) the energies and absolute values of the three-momenta are given by

$$
\begin{aligned}
E_q &= \frac{1}{2\sqrt{W^2}}\left[W^2 - Q^2 - M^2\right] \ , && \text{(A.12)}\\
E_{p_1} &= \frac{1}{2\sqrt{W^2}}\left[W^2 + Q^2 + M^2\right] \ , && \text{(A.13)}\\
|\boldsymbol{q}| &= |\boldsymbol{p}_1| = \frac{1}{2\sqrt{W^2}}\lambda^{1/2}(W^2, -Q^2, M^2) \ , && \text{(A.14)}\\
E_r &= \frac{1}{2\sqrt{W^2}}\left[W^2 + M_X^2 - M_1^2\right] \ , && \text{(A.15)}\\
E_{p_2} &= \frac{1}{2\sqrt{W^2}}\left[W^2 + M^2 - M_X^2\right] \ , && \text{(A.16)}\\
|\boldsymbol{r}| &= |\boldsymbol{p}_2| = \frac{1}{2\sqrt{W^2}}\lambda^{1/2}(W^2, M^2, M_X^2) \ , && \text{(A.17)}\\
E_l &= |\boldsymbol{l}| = \frac{1}{2\sqrt{W^2}}\left[s - Q^2 - M^2\right] \ . && \text{(A.18)}
\end{aligned}
$$

The spin vector $S_{||}$ and the four vector p_2 read

$$
\begin{aligned}
S_{||} &= \frac{1}{2\sqrt{W^2}}\left(\lambda^{1/2}(W^2, -Q^2, M^2); 0, 0, W^2 + M^2 + Q^2\right) \ , && \text{(A.19)}\\
p_2 &= \frac{1}{2\sqrt{W^2}}\left(W^2 + M^2 - M_X^2; \boldsymbol{p}_{\perp,2}, \cos\theta_{1,2}\lambda^{1/2}(W^2, M^2, M_X^2)\right) \ , && \text{(A.20)}
\end{aligned}
$$

with $S_{||}^2 = -M^2$ and

$$
\begin{aligned}
\cos\theta_{1,2} &= \frac{2W^2(t-2M^2)+(W^2+Q^2-M^2)(W^2+M^2-M_X^2)}{\sqrt{\lambda(W^2,M^2,-Q^2)\lambda(W^2,M^2,M_X^2)}} \\
&= \frac{1-x_{\mathbb{P}}+\frac{tx_{\rm BJ}}{Q^2}-\frac{4x_{\rm BJ}M^2}{Q^2}\left(1-x_{\rm BJ}+x_{\rm BJ}\frac{M^2}{Q^2}\right)-2\frac{x_{\rm BJ}^2 t}{Q^2}\left(1-\frac{M^2}{Q^2}\right)}{\left\{\left(1+\frac{4x_{\rm BJ}^2M^2}{Q^2}\right)\left[\left(1-x_{\mathbb{P}}-\frac{x_{\rm BJ}t}{Q^2}\right)^2-4x_{\rm BJ}x_{\mathbb{P}}\frac{M^2}{Q^2}\left(1-\beta+\beta\frac{t}{Q^2}\right)\right]\right\}} \\
&\simeq 1-\frac{x_{\rm BJ}}{1-x_{\mathbb{P}}}\left[\frac{|t|}{Q^2}\left(1+\frac{2}{1-x_{\mathbb{P}}}\right)+\frac{4M^2}{Q^2}\right]+\mathcal{O}\left(\left(x_{\rm BJ}^2,\frac{x_{\rm BJ}\mu^2}{Q^2}\right)^2\right)\ , \qquad \text{(A.21)}
\end{aligned}
$$

with $\mu^2 = t, M^2$. Note that the dependence on μ^2/Q^2 is here *linear* with $x_{\rm BJ}$.

$$
\lambda(a,b,c) = (a-b-c)^2 - 4bc \qquad \text{(A.22)}
$$

denotes the triangle-function. In the limit $t, M^2 \to 0$ one obtains $\cos\theta_{1,2} = 1$.

$S_{||}.p_2$ is given by

$$
\begin{aligned}
S_{||}.p_2 &= \frac{1}{4W^2}\Big[\lambda^{1/2}(W^2,-Q^2,M^2)(W^2+M^2-M_X^2) \\
&\qquad -\cos(\theta_{1,2})\lambda^{1/2}(W^2,M^2,M_X^2)(W^2+M^2+Q^2)\Big] \\
&= \frac{M^2x_{\mathbb{P}}(1-x_{\mathbb{P}}/2)}{1-x_{\mathbb{P}}}+\frac{|t|(3-x_{\mathbb{P}})}{4(1-x_{\rm BJ})(1-x_{\mathbb{P}})}+\mathcal{O}(|t|^2,M^4,|t|M^2)\ . \qquad \text{(A.23)}
\end{aligned}
$$

Further $S_{||}.l_1$ and $S_{||}.q$ are

$$
\begin{aligned}
S_{||}.l_1 &= \frac{1}{4W^2}\left(s-Q^2-M^2\right)\frac{Q^2}{x_{\rm BJ}}\left[\left(1+\frac{4x_{\rm BJ}^2M^2}{Q^2}\right)^{1/2}-\left(1+\frac{2x_{\rm BJ}M^2}{Q^2}\right)\right] \\
&\simeq -\frac{1}{2y}(1-x_{\rm BJ}y)M^2+\mathcal{O}\left(\frac{x_{\rm BJ}^2M^4}{Q^2}\right)\ , \qquad \text{(A.24)} \\
S_{||}.q &= -\frac{1}{2W^2}\left(Q^2-M^2\right)\frac{Q^2}{x_{\rm BJ}}\sqrt{1+\frac{4x_{\rm BJ}^2M^2}{Q^2}} \\
&= -\frac{Q^2}{1-x_{\rm BJ}}\left[1-\frac{M^2}{Q^2}\left(\frac{1}{1-x_{\rm BJ}}-4x_{\rm BJ}^2\right)+\mathcal{O}\left(\left(x_{\rm BJ}\frac{M^2}{Q^2}\right)^2\right)\right]\ . \qquad \text{(A.25)}
\end{aligned}
$$

Note that these expressions contain terms of $\mathcal{O}(M^2/Q^2)$ and $\mathcal{O}(x_{\rm BJ}M^2/Q^2)$. $S_{||}.l_1$ and $S_{||}.p_2$ vanish in the strict collinear limit $t, M^2 \to 0$.

The above invariants, except s_1, were all parameterized in terms of the dimensionless quantities, as $x_{\rm BJ}, y, x_{\mathbb{P}}$ keeping M^2 and t, which are normalized to Q^2. The invariant

$$
s_1 = s + M^2 - \frac{1}{\lambda(W^2,q^2,M^2)}\left[D_1 + 2\cos(\phi_b)\sqrt{G_1G_2}\right]\ , \qquad \text{(A.26)}
$$

in addition depends on the azimuthal angle ϕ_b. Here,

$$
\begin{aligned}
G_1 &= G(s,q^2,W^2,0,M^2,0) \le 0\ , \qquad \text{(A.27)} \\
G_2 &= G(W^2,t,M^2,q^2,M^2,M_X^2) \le 0\ , \qquad \text{(A.28)}
\end{aligned}
$$

where G denotes the Caley determinant

$$G(x,y,z,u,v,w) = -\frac{1}{2}\begin{vmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & v & x & z \\ 1 & v & 0 & u & y \\ 1 & x & u & 0 & w \\ 1 & z & y & w & 0 \end{vmatrix} . \tag{A.29}$$

D_1 is the determinant

$$D_1 = \begin{vmatrix} 2M^2 & W^2 - q^2 + M^2 & 2M^2 - t \\ W^2 - q^2 + M^2 & 2W^2 & W^2 - M_X^2 + M^2 \\ s + M^2 & s + W^2 & 0 \end{vmatrix} . \tag{A.30}$$

Let us consider the limit $M^2, t \to 0$. Here

$$G_2 = G(W^2, 0, 0, q^2, 0, M_X^2) = 0 , \tag{A.31}$$

and s_1 does not depend on the azimuthal angle ϕ_b. Furthermore,

$$D_1 = (W^2 + Q^2)(W^2 - M_X^2)s = \frac{sQ^4}{x_{\rm BJ}^2}(1 - x_{\mathbb{P}}) , \tag{A.32}$$

$$2l_1.p_2 = s(1 - x_{\mathbb{P}}) . \tag{A.33}$$

Therefore we obtain in the limit $M^2, t \to 0$ the hadronic tensors given in [2, 3].

We now expand $2l_1.p_2$ up to terms linear in M^2 and t. One obtains

$$G_1 \simeq -s^2 Q^2 \left[(1-y) - \frac{M^2}{Q^2} x_{\rm BJ} y \left(2 - y(1-x)\right)\right] , \tag{A.34}$$

$$G_2 \simeq -\frac{Q^4}{x_{\rm BJ}^2}\left[(1 - 2x_{\rm BJ} - x_{\mathbb{P}})\frac{|t|}{Q^2} - \frac{M^2}{Q^2} x_{\mathbb{P}}^2 (1 + 2\beta)^2\right] , \tag{A.35}$$

$$\begin{aligned} D_1 \simeq\ & \frac{Q^6}{y x_{\rm BJ}^3}\Big[1 - x_{\mathbb{P}} + \frac{|t|}{Q^2} x_{\rm BJ}(y(1 - x_{\rm BJ}) + 2x_{\rm BJ}) + 2\frac{M^2}{Q^2} x_{\rm BJ}(2x_{\rm BJ} + y(1 - x_{\rm BJ} x_{\mathbb{P}})) \\ & + \mathcal{O}(\mu^4/Q^4)\Big] . \end{aligned} \tag{A.36}$$

The ratio $l_1.p_2/l_1.p_1$ receives $\sqrt{\mu^2/Q^2}$ corrections for the angular term $\propto \cos\phi_b$ and μ^2/Q^2 corrections otherwise,

$$\begin{aligned} \frac{l_1.p_2}{l_1.p_1} =\ & 1 - x_{\mathbb{P}} + \frac{|t|}{Q^2} x_{\rm BJ}\left[y(1 - x_{\rm BJ}) + 2x_{\rm BJ}\right] + 2\frac{M^2}{Q^2} x_{\rm BJ} y (1 - x_{\rm BJ} x_{\mathbb{P}}) \\ & + 2\cos\phi_b x\sqrt{1-y}\left[(1 - 2x_{\rm BJ} - x_{\mathbb{P}})\frac{|t|}{Q^2} - \frac{M^2}{Q^2} x_{\mathbb{P}}^2 (1 + 2\beta)^2\right]^{1/2} \\ & + \mathcal{O}\left(\frac{\mu^3}{(Q^2)^{3/2}}\right) . \end{aligned} \tag{A.37}$$

B The Limiting Case of Deep–Inelastic Scattering

As a check of our general result we perform the limit $p_2 \to 0$ to obtain the results of Refs. [15–17]. In this limit the kinematic variables and invariants are given by

$$\mathcal{P} \to p_1 \equiv p\,, \qquad x \to 2x_{\rm Bj}^f \equiv \frac{Q^2}{qp}\,, \qquad \eta \to -1\,, \qquad \pi_- \to 0\,, \tag{B.1}$$

$$\mathcal{P}^2 \to M^2\,, \quad t \to M^2\,, \quad p_- p_+ \to -M^2\,, \qquad \mathcal{K}^1 \to p\,, \qquad \mathcal{K}^2 \to 0\,. \tag{B.2}$$

The generalized Nachtmann variable takes the form

$$\xi \;\to\; 2\,\frac{2x_{\rm Bj}^f}{1+\sqrt{1+4x_{\rm Bj}^f M^2/Q^2}} = 2\xi^f\,. \tag{B.3}$$

First, we consider the symmetric part of the amplitude. The second kinematic variable $\mathcal{K}^2 = \pi_-$ vanishes. In (4.17) only the contributions for $a = 1$ remain,

$$\mathsf{Im}\, T^{\rm tw2}_{1\,\{\mu\nu\}}(q) = 2\pi \int d\zeta \left[-\, g^{\rm T}_{\mu\nu}\, W^{\rm diff}_{1\,1}(x, \mathcal{P}^2/Q^2;\zeta) + \frac{p^{\rm T}_\mu p^{\rm T}_\nu}{M^2}\, W^{\rm diff}_{1\,2}(x, \mathcal{P}^2/Q^2;\zeta)\right] \tag{B.4}$$

$$\to 2\pi \left[-\, g^{\rm T}_{\mu\nu}\, W_1(\xi^f) + \frac{p^{\rm T}_\mu p^{\rm T}_\nu}{M^2}\, W_2(\xi^f)\right]. \tag{B.5}$$

Because of $p_2 = 0$, the integration over z_2 can now be performed,

$$\int dz_2\, \phi(z_1, z_2) = \widehat{\phi}(z_1)\;, \tag{B.6}$$

where $\widehat{\Phi}(z_1)$ denotes the parton density in the deep–inelastic case. The variables z_i are expressed by

$$z_1 \to \lambda = \xi\,, \quad z_2 \to \lambda(2\zeta+1) = \xi(2\zeta+1)\,, \quad dz_2 = 2\,\xi\, d\zeta\,. \tag{B.7}$$

From the complete integration measure $2|\lambda| d\lambda d\zeta$ the λ-integral has already been carried out, so that only the ζ-integration remains.

To get the standard structure functions for deep–inelastic scattering we take the limits (B.1–B.3) and perform the ζ-integration,

$$W_k(\xi^f, x_{\rm Bj}^f, p^2/Q^2) = \int d\zeta \lim_{p_2\to 0} W^{\rm diff}_{1\,k}(\xi, x, \mathcal{P}^2/Q^2;\zeta) \qquad \text{for} \qquad k = 1, 2\,. \tag{B.8}$$

To obtain explicit expressions we use $W^{\rm diff}_{1\,1}(\xi, x, \mathcal{P}^2/Q^2;\zeta)$ and $W^{\rm diff}_{1\,2}(\xi, x, \mathcal{P}^2/Q^2;\zeta)$ in (4.18) and (4.20) together with the diffractive structure functions $F_{1\,1}(\xi,\zeta)$ and $F_{1\,2}(\xi,\zeta)$ as given by (4.3) and (4.4), respectively. We obtain

$$W_1 = \frac{2\, x_{\rm Bj}^f}{\sqrt{1+4(x_{\rm Bj}^f)^2 M^2/Q^2}} \left[\widehat{\Phi}^{(0)}_{f\,1} + \frac{x_{\rm Bj}^f M^2/Q^2}{\sqrt{1+4(x_{\rm Bj}^f)^2 M^2/Q^2}}\, \widehat{\Phi}^{(1)}_{f\,1} + \frac{(x_{\rm Bj}^f\, M^2/Q^2)^2}{1+4(x_{\rm Bj}^f)^2 M^2/Q^2}\, \widehat{\Phi}^{(2)}_{f\,1} \right] \tag{B.9}$$

and

$$W_2 = \frac{(2\, x_{\rm Bj}^f)^3 M^2/Q^2}{\sqrt{1+4(x_{\rm Bj}^f)^2 M^2/Q^2}^{\,3}} \left[\widehat{\Phi}^{(0)}_{f\,1} + \frac{3\, x_{\rm Bj}^f\, M^2/Q^2}{\sqrt{1+4(x_{\rm Bj}^f)^2 M^2/Q^2}} \widehat{\Phi}^{(1)}_{f\,1} + \frac{3\,(x_{\rm Bj}^f\, M^2/Q^2)^2}{1+4(x_{\rm Bj}^f)^2 M^2/Q^2} \widehat{\Phi}^{(2)}_{f\,1} \right], \tag{B.10}$$

where the functions $\widehat{\Phi}^{(n)}_{f\,1}(2\xi^f)$ follow from (4.8–4.10).

The ζ–integrals can be performed taking into account

$$\widehat{\Phi}^{(n)}_{f\,1}(2\xi^f) = 2^n\, \Phi^{(n)}_{f\,1}(\xi^f), \tag{B.11}$$

which yields

$$\int d\zeta \Phi^{(0)}_a(\xi,\zeta) \equiv \widehat{\Phi}^{(0)}_a(\xi) \to \widehat{\Phi}^{(0)}_{f\,a}(2\xi^f) = f_{f\,a}(\xi^f) \; = \Phi^{(0)}_{f\,a}(\xi^f)\,, \tag{B.12}$$

$$\int d\zeta \Phi^{(1)}_a(\xi,\zeta) \equiv \widehat{\Phi}^{(1)}_a(\xi) \to \widehat{\Phi}^{(1)}_{f\,a}(2\xi^f) = 2 \int_{\xi^f}^1 dy_1\; f_{f\,a}(y_1) = 2\Phi^{(1)}_{f\,a}(\xi^f)\,, \tag{B.13}$$

$$\int d\zeta \Phi^{(2)}_a(\xi,\zeta) \equiv \widehat{\Phi}^{(2)}_a(\xi) \to \widehat{\Phi}^{(2)}_{f\,a}(2\xi^f) = 4 \int_{\xi^f}^1 dy_2 \int_{y_2}^1 dy_1\; f_{f\,a}(y_1) = 4\Phi^{(2)}_{f\,a}(\xi^f)\,, \tag{B.14}$$

in the limit $p_2 \to 0$. Finally one obtains

$$W_1(\xi^f) = \frac{2\, x_{\rm Bj}^f}{\sqrt{1+4(x_{\rm Bj}^f)^2 M^2/Q^2}} \left[\Phi^{(0)}_{f\,1} + \frac{2\, x_{\rm Bj}^f M^2/Q^2}{\sqrt{1+4(x_{\rm Bj}^f)^2 M^2/Q^2}} \Phi^{(1)}_{f\,1} + \frac{4\,(x_{\rm Bj}^f\, M^2/Q^2)^2}{1+4(x_{\rm Bj}^f)^2 M^2/Q^2} \Phi^{(2)}_{f\,1} \right] \tag{B.15}$$

and

$$W_2(\xi^f) = \frac{(2\, x_{\rm Bj}^f)^3 M^2/Q^2}{\sqrt{1+4(x_{\rm Bj}^f)^2 M^2/Q^2}^{\,3}} \left[\Phi^{(0)}_{f\,1} + \frac{6\, x_{\rm Bj}^f M^2/Q^2}{\sqrt{1+4(x_{\rm Bj}^f)^2 M^2/Q^2}} \Phi^{(1)}_{f\,1} + \frac{12\,(x_{\rm Bj}^f\, M^2/Q^2)^2}{1+4(x_{\rm Bj}^f)^2 M^2/Q^2} \Phi^{(2)}_{f\,1} \right], \tag{B.16}$$

the representation for the target mass corrections in the unpolarized case given in [15] before.

As in the case of generalized parton densities also here the diffractive hadronic distribution amplitudes contain as limit the parton distribution of deeply inelastic scattering. However, care is needed because

$$\Phi^{(0)}_{f\,a}(\xi_f) = \widehat{\Phi}_a(2\xi^f, t = M^2) \tag{B.17}$$

includes an analytic continuation from the physical values $t < 0$ to $t = M^2$.

Next, we consider the antisymmetric contributions in the Compton amplitude, which correspond to the case of polarized scattering. From the kinematic factors (3.8) only $\mathcal{K}^1_5 = S$ remains in the limit $p_2 \to 0$. We consider (4.62) and (4.64) with the definition (4.61) for $G^{(n)}_{1\,k}$. This results in

$$\mathrm{Im}\, T^{\rm tw2}_{[\mu\nu]\,f}\,(q) \;\; = \;\; \pi\, \epsilon_{\mu\nu}{}^{\alpha\beta} \left\{ \frac{q_\alpha S_\beta}{qp} \Big(G^{(0)}_{11}(x_{\rm Bj}^f) + G^{(0)}_{12}(x_{\rm Bj}^f) \Big) - \frac{q_\alpha p_\beta}{qp}\, \frac{qS}{qp}\, G^{(0)}_{12}(x_{\rm Bj}^f) \right\}, \tag{B.18}$$

the forward scattering limit (B.1) of our general result (4.62) and (4.64) with the definition (4.61) of $G_{1\,k}^{(n)}$. The antisymmetric part of the amplitude simplifies to

$$G_{11}^{(0)}(x_{\rm Bj}^f) = \frac{x_{\rm Bj}^f/\xi^f}{[1+4(x_{\rm Bj}^f)^2M^2/Q^2]^{3/2}}\Bigg[\widehat{\Phi}_{51}^{(0)}(2\xi^f) + \frac{4x_{\rm Bj}^f(x_{\rm Bj}^f+\xi^f)M^2/Q^2}{[1+(4x_{\rm Bj}^f)^2M^2/Q^2]^{1/2}}\,\widehat{\Phi}_{51}^{(1)}(2\xi^f)$$
$$-\,2x_{\rm Bj}^f\xi^fM^2/Q^2\frac{2-4(x_{\rm Bj}^f)^2M^2/Q^2}{1+4(x_{\rm Bj}^f)^2M^2/Q^2}\,\widehat{\Phi}_{51}^{(2)}(2\xi^f)\Bigg], \tag{B.19}$$

$$G_{12}^{(0)}(x_{\rm Bj}^f) = \frac{-x_{\rm Bj}^f/\xi^f}{[1+4(x_{\rm Bj}^f)^2M^2/Q^2]^{3/2}}\Bigg[\widehat{\Phi}_{51}^{(0)}(2\xi^f) - \frac{1-4x_{\rm Bj}^f\xi^fM^2/Q^2}{[1+4(x_{\rm Bj}^f)^2M^2/Q^2]^{1/2}}\,\widehat{\Phi}_{51}^{(1)}(2\xi^f)$$
$$-\,\frac{6\,x_{\rm Bj}^f\xi^fM^2/Q^2}{1+4(x_{\rm Bj}^f)^2M^2/Q^2}\,\widehat{\Phi}_{51}^{(2)}(2\xi^f)\Bigg], \tag{B.20}$$

and

$$G_{11}^{(0)}(x_{\rm Bj}^f) + G_{12}^{(0)}(x_{\rm Bj}^f) = \frac{x_{\rm Bj}^f/\xi^f}{[1+4(x_{\rm Bj}^f)^2M^2/Q^2]^{3/2}}$$
$$\times\,\Big[\Big(1+2x_{\rm Bj}^f\xi^fM^2/Q^2\Big)\,\widehat{\Phi}_{51}^{(1)}(2\xi^f)\,+2x_{\rm Bj}^f\xi^fM^2/Q^2\,\widehat{\Phi}_{51}^{(2)}(2\xi^f)\Big]. \tag{B.21}$$

Again we introduced the (integrated) parton distributions $\widehat{\Phi}_{5\,a}^{(0)}(\xi)$ and performed the limit (B.1) as follows:

$$\int d\zeta\Phi_{5\,a}^{(0)}(\xi,\zeta) \;\equiv\; \widehat{\Phi}_{5\,a}^{(0)}(\xi) \to \widehat{\Phi}_{5\,a}^{(0)}(2\xi^f) = 2\xi^f f_{5\,f\,a}(\xi^f) \;= \Phi_{5\,f\,a}^{(0)}(\xi^f)\,, \tag{B.22}$$

$$\int d\zeta\Phi_{5\,a}^{(1)}(\xi,\zeta) \;\equiv\; \widehat{\Phi}_{5\,a}^{(1)}(\xi) \to \widehat{\Phi}_{5\,a}^{(1)}(2\xi^f) = \int_{\xi^f}^1\frac{dy_1}{y_1}\,\Phi_{5\,f\,a}^{(0)}(y_1) = \Phi_{5\,f\,a}^{(1)}(\xi^f)\,, \tag{B.23}$$

$$\int d\zeta\Phi_{5\,a}^{(2)}(\xi,\zeta) \;\equiv\; \widehat{\Phi}_{5\,a}^{(2)}(\xi) \to \widehat{\Phi}_{5\,a}^{(2)}(2\xi^f) = \int_{\xi^f}^1\frac{dy_2}{y_2}\int_{y_2}^1\frac{dy_1}{y_1}\,\Phi_{5\,f\,a}^{(0)}(y_1) = \Phi_{5\,f\,a}^{(2)}(\xi^f)\;. \tag{B.24}$$

Finally we substitute $\widehat{\Phi}_{5\,a}^{(i)}(2\xi^f)$ by $\Phi_{5\,f\,a}^{(2)}(\xi^f)$ and obtain the result given in [16, 17] before.

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